STOCHASTIC ANALYSIS OF N-UNIT PLANT WITH DIFFERENT TYPES OF FAILURE AND REPAIR POLICY USING COPULA DISTRIBUTION

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Abstract
This paper deals with the reliability and profit analysis of an N-unit plant with two types of failure i.e. Partial failure and complete failure. The plant has N-units and at least k units are required to be in operational mode to fulfill the satisfactory needs of customer demand. Till the time k units are in good operational condition, i.e. at most (k-1) units fail, the plant can fulfill the needs of market demands and failure of more than k units it cannot satisfy customer requirements. Other types of failures like human failure and deliberate failure can also affect the functioning of plant. An important aspect of system failure, which is usually observed in industry, the strike of employees is also considered for study of mathematical model. The natural calamities such as earthquake, tsunami and sudden catching of fire that can fail the system completely, have also been considered in study of the model. The system has been studied by supplementary variable technique and various measures of reliability, such as availability, state transition probabilities, MTTF and profit function have been discussed. Some particular cases have also been discussed by taking different values of failure rates.

Key Words: Controller Failure, Deliberate Failure, Gumbel-Hougaard Family Copula, MTTF, Natural Calamity, Strike, Supplementary Variables.

1. Introduction
Reliability plays an important role in industrial automation. A high-level reliability is being demanded everywhere in industry and organizations. In this aspect, the various researchers have done a remarkable work in designing the systems for future demands. Redundancy is one of the techniques, which improves the reliability of system. The k-out-of-n system is an arrangement of unit in parallel configuration in such a way that the entire system performs the task until the k units are in good working condition. Only one operational unit of any industrial organization can not fulfill the necessity of market demand. In real life situations, it is observed that after some time the operational unit has to shut down for time to reduce the negative effect like heating, cooling and maintenance of oiling etc. Moreover, continuously running a single unit may have adverse effect and has a possibility of damaging some important part of system. Therefore, it is necessary to design the system, which consists of some unit in standby mode that can perform the task whenever it is required. Consecutive k-out-of-n systems have been studied by Kontoleon [8] and Ramamuthy [14] who proposed explicit formulas for k-out-of-n systems. A consecutive-k-out-of-n: F-system consists of n
ordered components along a line or circle such that the system fails if at least k consecutive components fail.

Many researchers have paid their attention to evaluate reliability and availability of repairable k-out-of-n: good and k-out-of-n: fail systems. A, k-out-of-n system is found to have extensive applications in many fields of engineering and science, including power plants, airplane model with duet engine and industrial organizations model. Xiaolin Liang et al. [21] studied a consecutive k-out-of-n repairable system under the concept of k-component, or ordinary component such that the priority of repair assigned for k components. The authors [19] have studied the operational behavior of multi state- state k-out-of-n: type analysis of system by considering 2-out-of-3: G, system as a special case for computations. The study of Ibrahim Yusuf et al. [7], focused on the comparative study of 2-out – of – 3: systems for different situations under the concept of general repair. So many researchers including [1, 3, 13, 19, 20, 22] have studied the complex systems by considering the fact that the failed system can be repaired by general repair policy. They studied the reliability characteristic of complex systems under the fact that only one repair can be employed between two-transition states, but there are many situations where more than one repair is required between two adjacent transition states. When such possibility is observed, the system is studied using copula [12]. The authors [16] have also studied the complex systems having three units-super priority, priority and ordinary, under preemptive resume repairs policy. Alka Munjal et al. [2] have studied a complex repairable system composed of a 2-out-of-3: G; subsystems connected in parallel configuration. In modern competitive world, advance technology has introduced many electronic devices, which can improve the efficiency of system. The controller is one of such electronic devices, which is widely used in many electronic systems and equipments. In consequence to the study of complex systems with controller the researchers including [17, 18] have discussed the reliability measures of a system, having two subsystems with controller, human and deliberate failure. Various researchers have extensively studied the standby redundant complex systems. In reference to the study of standby redundant complex systems, Singh et al. [11] have studied availability of standby complex system under waiting repair and human failure using Gumbel-Hougaard family copula distribution. Furthermore, number of researchers studied reliability measures and tried to obtain best structure of field to fulfill the challenges of industry requirements, but still more attention is required in this field.

Considering necessity and requirements of future prospects the study of such type of systems, the authors have described a mathematical model of N-unit plant, which works under k-out-of-n: good policy. Initially in state S0 the system is in perfect state, where all units are in good working condition. When the system starts functioning, due to failure of some (k - i), i= 1, 2, 3..., units in the system, it approaches to the degraded state S1. Further failure of more than k units, the system approaches to completely failed state S2. Human and deliberate failure brings the system in to complete failed states S3 and S6 respectively. An automatic controller, which controls the functioning of the plant should be considered as important phase of system failure. Controller failure brings the system into complete failed state S8. In real life experience, strike of employees is a routine phenomenon seen in industry, which affects the functioning of plant/ organization. The state S4 and S5 represents when employees strike
appears and then the system has to wait for further operation. The natural calamities such as earthquake, tsunami etc which may completely damage the system are represented by the state $S_6$. Whenever the system comes in degraded mode, general repair may be employed but in completely failed states of the system repair can be accomplished by Gumbel- Hougaard family copula distribution. The system is analyzed by supplementary variable technique, various measures of reliability have also been discussed and some particular cases are taken to highlight the results. Results and conclusions are described by Tables and graphs.

The entire paper has been divided into following sections; Section I of paper is introduction, which consists the related work done by previous researchers and need of study is highlighted. Section II of paper is state description for clearly understanding the description of various states. Section III consists of the basic assumptions for elaborated model. In section, IV and V are the state transition diagram and notations that have been used for mathematical modeling and solution respectively. In section VI, paper covers mathematical modeling and solution of formulated model. The VII section of the paper is an analytical part in which the various measures of reliability like availability, MTTF and profit analysis have been evaluated for different values of parameters. The last VII section of paper includes conclusion part for discussion of results and their explanation for future prospects.

2. State Description

<table>
<thead>
<tr>
<th>State</th>
<th>State Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>All units of system are good working condition and system is in perfect state.</td>
</tr>
<tr>
<td>$S_1$</td>
<td>The state $S_1$ represents that, at least k unit of system are in good working condition.</td>
</tr>
<tr>
<td>$S_2$</td>
<td>In $S_2$ state the system has fail due to failure of $(k + i)$ units of system. $i = 1, 2, 3, \ldots \ldots \ldots \ldots$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>The state $S_3$ represents appearance of human failure due to which, the system approaches to complete failure mode.</td>
</tr>
<tr>
<td>$S_4$</td>
<td>In state $S_4$ the system is in degraded state due to sudden strike of employees.</td>
</tr>
<tr>
<td>$S_5$</td>
<td>State $S_5$ represents the state where system is suppose to wait for further functioning due to strike.</td>
</tr>
<tr>
<td>$S_6$</td>
<td>In state $S_6$ the system has been failed due to deliberate failure by operator.</td>
</tr>
<tr>
<td>$S_7$</td>
<td>The system fails completely due to failure of natural calamity that appears in state $S_7$.</td>
</tr>
<tr>
<td>$S_8$</td>
<td>System fails completely due to controller failure, which may be seen in state $S_8$.</td>
</tr>
</tbody>
</table>
3. Assumptions

The following assumptions are taken throughout the discussion of model.

(1) Initially the system is in perfect state $S_0$ and all units are in good working condition.

(2) The system works successfully until at least $k$- units of it is in good working condition.

(3) System fails if more than $k$ units fail.

(4) Only one change is allowed at a time in the transitions.

(5) The strike of employees stop functioning of system and due to strike the system waits for restarts functioning.

(6) Deliberate failure, human failure as well as natural calamity completely fail the system.

(7) Partially failed states have been repaired by employing general time distribution.

(8) Deliberate failure / human failure / natural calamity and controller failure system need fast repairing and hence they are repaired by using (Gumbel-Hougaard) family copula distribution.

(9) Repaired system works like a new and repair did not damage anything.

4. State Transition Diagram of Model

![State Transition Diagram of Model](image-url)
5. Notations

\( \lambda_1 / \lambda_2 \)  
Failure rates of system such that at most k unit /more than k units failed during operational mode.

\( \lambda_D / \lambda_{CL} / \lambda_C / \lambda_S / \lambda_h \)  
Failure rate of deliberate failure/ failure due to natural calamities/ controller failure/ failure due to strike/ human failure.

\( \phi(x) \)  
General repair rate of system for degraded state i.e. state S_i and S_5.

\( P_i(t) \)  
Represents state transition probabilities of respective states of the system.

\( \overline{P}(s) \)  
Laplace transform of state transition probabilities P(t).

\( P_i(x, t) \)  
State transition probability that the system is in state S_i, i=1,.... 8. P_i(x, t) represents that the system in state i. The system is in repair with repair variables x, t.

\( \exp[x + \{\log(\phi(x))\}^\phi] \)  
The expression for joint probability distribution (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula for given \( \phi(x) = \) general repair, e^x = exponential repair rate and coupling both gives \( \exp[x + \{\log(\phi(x))\}^\phi] = \Theta(x) e^x \)

\( E_{\phi}(t) \)  
Expected profit in interval [0,t)

6. Formulation of Mathematical Model

By probability considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model.

\[
\begin{align*}
\frac{\partial}{\partial t} + \lambda_1 + \lambda_{CL} + \lambda_C + \lambda_h + \lambda_D + \lambda_S \bigg[ P_0(t) &= \int_0^\infty \phi(x)P_1(x,t)dx + \int_0^\infty \exp[y + \{\log(\phi(y))\}^\phi] P_0(y,t)dy \\
&+ \int_0^\infty \exp[x + \{\log(\phi(x))\}^\phi] P_5(x,t)dx + \int_0^\infty \exp[x + \{\log(\phi(x))\}^\phi] P_1(x,t)dx \\
&+ \int_0^\infty \exp[z + \{\log(\phi(z))\}^\phi] P_6(z,t)dz + \int_0^\infty \phi(x)P_3(x,t)dx + \int_0^\infty \exp[y + \{\log(\phi(y))\}^\phi] P_3(y,t)dy
\end{align*}
\]

\[
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_C + \lambda_{CL} + \lambda_D + \lambda_h + \lambda_S + \phi(x) \right] P_1(x,t) = 0 \tag{6.1}
\]

\( P_1(x,t) = 0 \)
\[
\begin{align*}
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \exp\left[ x^\theta + \{\log \phi(x)\}^\theta \right] P_2(x,t) &= 0 \tag{6.3} \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial y} \exp\left[ y^\theta + \{\log \phi(y)\}^\theta \right] P_3(y,t) &= 0 \tag{6.4} \\
\frac{\partial}{\partial t} + \lambda_p + \lambda_c + \lambda_{cl} + \phi(x) P_4(x,t) &= 0 \tag{6.5} \\
\frac{\partial}{\partial t} + \lambda_{cl} + \lambda_c + \lambda_d + \lambda_n + \phi(x) P_5(x,t) &= 0 \tag{6.6} \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial z} \exp\left[ z^\theta + \{\log \phi(z)\}^\theta \right] P_6(z,t) &= 0 \tag{6.7} \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \exp\left[ x^\theta + \{\log \phi(x)\}^\theta \right] P_7(x,t) &= 0 \tag{6.8} \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial y} \exp\left[ y^\theta + \{\log \phi(y)\}^\theta \right] P_8(y,t) &= 0 \tag{6.9}
\end{align*}
\]

**Boundary Conditions**

\[
\begin{align*}
P_1(0,t) &= \lambda_1 P_0(t) \tag{6.10} \\
P_2(0,t) &= \lambda_1 \lambda_2 P_0(t) \tag{6.11} \\
P_3(0,t) &= \lambda_3 P_0(t) + \lambda_3 P_3(0,t) + \lambda_3 P_3(0,t) \tag{6.12} \\
P_4(t) &= \lambda_5 P_0(t) + \lambda_5 P_1(0,t) \tag{6.13} \\
P_5(t) &= \lambda_n P_4(0,t) = \lambda_n \lambda_5 P_0(t) \tag{6.14} \\
P_6(0,t) &= \lambda_D P_0(t) + \lambda_D P_1(0,t) + \lambda_D P_3(0,t) \tag{6.15} \\
P_7(0,t) &= \lambda_{cl} P_0(t) + \lambda_{cl} P_1(0,t) + \lambda_{cl} P_4(0,t) + \lambda_{cl} P_4(0,t) \tag{6.16} \\
P_8(0,t) &= \lambda_c P_0(t) + \lambda_c P_1(0,t) + \lambda_c P_4(0,t) + \lambda_c P_4(0,t) \tag{6.17}
\end{align*}
\]

**Initial Conditions**

\[ P_0(0) = 1 \quad \text{and other state probabilities are zero at } t = 0 \tag{6.18} \]

**Solution of the Model**

Taking Laplace transformation of equations (6.1)-(6.17) and using equation (6.18), we obtain.
\begin{align}
(s + \lambda_1 + \lambda_{CL} + \lambda_c + \lambda_2 + \lambda_n + \lambda_y) \bar{P}_0(s) &= [1 + \int_0^s \bar{P}_1(x,s)dx + \int_0^s \bar{P}_2(y,s) \exp[\phi(x)]^{1\theta} dy + \int_0^s \bar{P}_3(z,s) \exp[\phi(x)]^{1\theta} dz]
\end{align}

\begin{align}
\int_0^s \bar{P}_1(x,s) \phi(x) dx + \int_0^s \bar{P}_2(y,s) \exp[\phi(x)]^{1\theta} dx + \int_0^s \bar{P}_3(z,s) \exp[\phi(x)]^{1\theta} dz
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial x} + \lambda_2 + \lambda_C + \lambda_D + \lambda_s + \lambda_n + \lambda_{CL} + \phi(x) \right] \bar{P}_1(x,s) = 0
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial x} + \exp[\phi(x)]^{1\theta} \right] \bar{P}_2(x,s) = 0
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial y} + \exp[\phi(y)]^{1\theta} \right] \bar{P}_3(y,s) = 0
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial x} + \lambda_n + \lambda_{CL} + \lambda_c + \phi(x) \right] \bar{P}_3(x,s) = 0
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial z} + \exp[\phi(z)]^{1\theta} \right] \bar{P}_6(z,s) = 0
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial x} + \exp[\phi(y)]^{1\theta} \right] \bar{P}_7(x,s) = 0
\end{align}

\begin{align}
\left[ s + \frac{\partial}{\partial y} + \exp[\phi(y)]^{1\theta} \right] \bar{P}_8(y,s) = 0
\end{align}

\begin{align}
\bar{P}_1(0,s) &= \lambda_1 \bar{P}_0(s)
\end{align}

\begin{align}
\bar{P}_2(0,s) &= \lambda_2 \bar{P}_0(s)
\end{align}

\begin{align}
\bar{P}_3(0,s) &= \lambda_n \bar{P}_0(s) + \lambda_n \bar{P}_5(0,s) + \lambda_n \bar{P}_1(0,s)
\end{align}

\begin{align}
\bar{P}_4(0,s) &= \lambda_s (\bar{P}_0(s) + \bar{P}_1(0,s))
\end{align}

\begin{align}
\bar{P}_5(0,s) &= \lambda_w \bar{P}_0(s)
\end{align}

\begin{align}
\bar{P}_6(0,s) &= \lambda_D (\bar{P}_0(s) + \bar{P}_1(0,s) + \bar{P}_3(0,s))
\end{align}

\begin{align}
\bar{P}_7(0,s) &= \lambda_{CL} (\bar{P}_0(s) + \bar{P}_1(0,s) + \bar{P}_3(0,s) + \bar{P}_4(0,s))
\end{align}

\begin{align}
\bar{P}_8(0,s) &= \lambda_C (\bar{P}_0(s) + \bar{P}_1(0,s) + \bar{P}_3(0,s) + \bar{P}_4(0,s))
\end{align}

Solving (6.20)-(6.27) with the help of (6.28)-(6.35), one may get
The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

\[
\bar{P}_0(s) = \frac{1}{D(s)} \quad (6.36)
\]

\[
\bar{P}_1(s) = \frac{\lambda_1 (1 - S_{\beta}(s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_{CL} + \lambda_4 + \lambda_5 + \lambda_{CL}))}{D(s) (s + \lambda_2 + \lambda_3 + \lambda_{CL} + \lambda_4 + \lambda_5 + \lambda_{CL})} \quad (6.37)
\]

\[
\bar{P}_2(s) = \frac{\lambda_2 \lambda_3}{D(s)} \left(1 - \frac{S_{\exp[\beta (\log \phi(s))]^{\phi}}(s)}{s}\right) \quad (6.38)
\]

\[
\bar{P}_3(s) = \frac{\lambda_3 (1 + \lambda_1 + \lambda_4 + \lambda_5)}{D(s)} \left(1 - \frac{S_{\exp[\beta (\log \phi(s))]^{\phi}}(s)}{s}\right) \quad (6.39)
\]

\[
\bar{P}_4(s) = \frac{\lambda_4 (1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL})}{D(s) (s + \lambda_2 + \lambda_3 + \lambda_{CL})} \quad (6.40)
\]

\[
\bar{P}_5(s) = \frac{\lambda_5 (1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL})}{D(s) (s + \lambda_2 + \lambda_3 + \lambda_{CL} + \lambda_{CL} + \lambda_{CL})} \quad (6.41)
\]

\[
\bar{P}_6(s) = \frac{\lambda_6 (1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL})}{D(s)} \left(1 - \frac{S_{\exp[\beta (\log \phi(s))]^{\phi}}(s)}{s}\right) \quad (6.42)
\]

\[
\bar{P}_7(s) = \frac{\lambda_7 (1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL})}{D(s)} \left(1 - \frac{S_{\exp[\beta (\log \phi(s))]^{\phi}}(s)}{s}\right) \quad (6.43)
\]

\[
\bar{P}_8(s) = \frac{\lambda_8 (1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL})}{D(s)} \left(1 - \frac{S_{\exp[\beta (\log \phi(s))]^{\phi}}(s)}{s}\right) \quad (6.44)
\]

\[
D(s) = \left[\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \lambda_{CL} \left(1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL} \right) \right] + \left[\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \lambda_{CL} \left(1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL} \right) \right] + \left[\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \lambda_{CL} \left(1 + \lambda_1 + \lambda_4 + \lambda_5 + \lambda_{CL} \right) \right]
\]

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

\[
\bar{P}_\text{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_4(s) + \bar{P}_5(s)
\]
\[ P_{\text{failed}}(s) = P_1(s) + P_2(s) + P_3(s) + P_4(s) + P_5(s) \] (6.47)

### 7. Particular Cases

#### 7 A: Availability Analysis

When repair follows exponential time distribution;
Using the formulas

\[
\overline{S} \exp \left[ x^\theta + \{\log \phi(x)\}^\theta \right] (s) = \frac{\exp[ x^\theta + \{\log \phi(x)\}^\theta ]^{1/\theta}}{s + \exp[ x^\theta + \{\log \phi(x)\}^\theta ]^{1/\theta}}, \quad \overline{S}_\phi(s) = \frac{\phi}{s + \phi}
\]

in equation (6.46) and setting the values of different parameters as \( \lambda_1 = 0.05, \lambda_2 = 0.06, \lambda_C = 0.025, \lambda_D = 0.02, \lambda_{CL} = 0.015, \lambda_h = 0.012, \lambda_S = 0.01, \phi = 1, \theta = 1, x = 1, y = 1, z = 1 \) then taking inverse Laplace transform, one can obtain expression for availability of system as:

\[ P_{\text{up}}(t) = -0.005017342 e^{-2.79348t} + 0.0008933 e^{-1.20836t} + 0.00012558 e^{-0.0703313t} + 0.0226998 e^{-1.02314t} + 0.981298 e^{-0.002993939t} \] (7.1)

For, \( t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \ldots \) One may get different values of \( P_{\text{up}}(t) \) as shown in Table 1 and corresponding figure 2.

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>1</td>
<td>0.987</td>
</tr>
<tr>
<td>2</td>
<td>0.978</td>
</tr>
<tr>
<td>3</td>
<td>0.974</td>
</tr>
<tr>
<td>4</td>
<td>0.970</td>
</tr>
<tr>
<td>5</td>
<td>0.967</td>
</tr>
<tr>
<td>6</td>
<td>0.964</td>
</tr>
<tr>
<td>7</td>
<td>0.961</td>
</tr>
<tr>
<td>8</td>
<td>0.958</td>
</tr>
<tr>
<td>9</td>
<td>0.955</td>
</tr>
</tbody>
</table>

**Table 1. Variation of Availability with respect to time**
7 B: Mean Time to Failure (MTTF)

Setting

\[ S_{\text{exp}}(x^r + \log(\phi(x))^\theta)^{1/\theta} = \frac{\exp[ x^\theta + \log(\phi(x))^\theta ]^{1/\theta}}{s + \exp[ x^\theta + \log(\phi(x))^\theta ]^{1/\theta}} \]

and taking all repairs to zero in equation (6.46). Taking limit, as \( s \) tends to zero one can obtain the MTTF as:

\[
MTTF = \frac{1}{(\lambda_1 + \lambda_C + \lambda_{CL} + \lambda_D + \lambda_h + \lambda_S)} \left[ \frac{\lambda_1}{(\lambda_2 + \lambda_C + \lambda_{CL} + \lambda_D + \lambda_h + \lambda_S)} + \frac{\lambda_S(1+\lambda_1)}{(\lambda_w + \lambda_C + \lambda_{CL})} \right]
\]

(7.2.1)

Setting, \( \lambda_2=0.06, \lambda_D=0.02, \lambda_h=0.01, \lambda_S=0.03, \lambda_{CL}=0.015, \lambda_C=0.025, \lambda_w=0.012, \) in above expression (7.2.1) and then varying \( \lambda_1 \) as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 respectively, one can obtain the values of MTTF with respect to failure rate \( \lambda_1 \).

The second column of Table 3 demonstrates the variation of MTTF with respect to failure rate \( \lambda_1 \).

Setting, \( \lambda_1=0.05, \lambda_D=0.02, \lambda_h=0.01, \lambda_S=0.03, \lambda_{CL}=0.015, \lambda_C=0.025, \lambda_w=0.012, \) and varying \( \lambda_2 \) as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) we can get values of MTTF corresponding to failure rate \( \lambda_2 \). Third column of Table 3 represents the values of failure rate corresponding to failure rate \( \lambda_2 \).

Setting \( \lambda_2=0.06, \lambda_1=0.05, \lambda_h=0.01, \lambda_S=0.03, \lambda_{CL}=0.015, \lambda_C=0.025, \lambda_w=0.012, \) and varying \( \lambda_D \) as: 0.01, 0.02, 0.03, 0.04, 0.005, 0.06, 0.07, 0.08, 0.09 in (7.2.1) we can obtain MTTF respect to failure rate \( \lambda_D \). Column 4 of Table 3 shows variation of MTTF with respect to \( \lambda_D \).
Setting $\lambda_2=0.06$, $\lambda_D=0.02$, $\lambda_1=0.05$, $\lambda_S=0.03$, $\lambda_{CL}=0.015$, $\lambda_C=0.025$, $\lambda_w=0.012$, and varying $\lambda_h$ as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one can obtain MTTF with respect to failure rate $\lambda_h$. Column 5 of Table 3 reveals variation of MTTF with respect to $\lambda_h$.

Setting $\lambda_2=0.06$, $\lambda_D=0.02$, $\lambda_1=0.05$, $\lambda_h=0.01$, $\lambda_{CL}=0.015$, $\lambda_C=0.025$, $\lambda_w=0.012$, and then varying $\lambda_S$ as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one obtain MTTF respect to failure rate $\lambda_S$. Column 6 of Table 3 shows variation of MTTF with respect to $\lambda_D$.

Setting $\lambda_2=0.06$, $\lambda_D=0.02$, $\lambda_1=0.05$, $\lambda_S=0.03$, $\lambda_h=0.015$, $\lambda_w=0.012$, and then varying $\lambda_{CL}$ as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one obtain MTTF with respect to $\lambda_{CL}$. The computed values of MTTF are shown in column7 of Table 3.

Setting $\lambda_2=0.06$, $\lambda_D=0.02$, $\lambda_1=0.05$, $\lambda_S=0.03$, $\lambda_{CL}=0.015$, $\lambda_C=0.025$, $\lambda_w=0.01$, and then varying $\lambda_h$ as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one can get MTTF values for the failure rate $\lambda_h$ in column9 in Table 3 given as under.

<table>
<thead>
<tr>
<th>Failure rate</th>
<th>MTTF $\lambda_1$</th>
<th>MTTF $\lambda_2$</th>
<th>MTTF $\lambda_D$</th>
<th>MTTF $\lambda_1$</th>
<th>MTTF $\lambda_2$</th>
<th>MTTF $\lambda_D$</th>
<th>MTTF $\lambda_h$</th>
<th>MTTF $\lambda_S$</th>
<th>MTTF $\lambda_{CL}$</th>
<th>MTTF $\lambda_C$</th>
<th>MTTF $\lambda_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>13.7452</td>
<td>13.3035</td>
<td>11.9024</td>
<td>11.1032</td>
<td>12.8227</td>
<td>10.6673</td>
<td>12.0030</td>
<td>11.8357</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>12.8227</td>
<td>12.9616</td>
<td>10.4029</td>
<td>9.7844</td>
<td>13.5057</td>
<td>8.7671</td>
<td>9.6180</td>
<td>11.2262</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td>11.8234</td>
<td>12.5913</td>
<td>8.7420</td>
<td>8.2988</td>
<td>14.3360</td>
<td>6.9494</td>
<td>7.4626</td>
<td>10.7286</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Computation of MTTF for different values of failure rates.
C: Cost Analysis

Let the failure rates of system be \( \lambda_1 = 0.05, \lambda_2 = 0.06, \lambda_C = 0.025, \lambda_D = 0.02, \lambda_{CL} = 0.015, \lambda_w = 0.012, \lambda_h = 0.01, \lambda_s = 0.03 \), mean time to repair of be \( \phi(x) = 1, x = 1, \theta = 1 \), and setting,

\[
\tilde{S}_{\text{out}, x^{\theta} + \log \phi(x)}^{\theta / \theta} (s) = \frac{\exp[ x^{\theta} + \{ \log \phi(x) \}^{\theta} \phi]^{1 / \theta}}{s + \exp[ (x^{\theta} + \{ \log \phi(x) \}^{\theta}) \theta]} \cdot \tilde{S}_\phi(s) = \frac{\phi}{s + \phi}
\]

equation (6.46) and taking inverse Laplace transform, one can obtain (7.3.1).

Let the service facility be always available, then expected profit during the interval \([0, t)\) is

\[
E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t
\]

Where \( K_1 \) and \( K_2 \) are revenue service cost per unit time. Hence

\[
E_p(t) = K_1 (0.001796e^{(-2.793487t)} - 0.000739e^{(-1.208364t)} - 0.000117e^{(-1.070313t)} - 0.022186e^{(-1.023140t)} - 327.755101e^{(-0.00129993t)} + 327.77634) - K_2 t
\]

Setting \( K_1 = 1 \) and \( K_2 = 0.5, 0.25, 20, 0.15, 0.10, \) and 05 respectively and varying \( t = 0.1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots \), one get Table 4, which give the expected profit with variation of time \( t \).
Table 3. Computation of Expected profit

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>$E_p(t)$: $K_2=0.05$</th>
<th>$E_p(t)$: $K_2=0.25$</th>
<th>$E_p(t)$: $K_2=0.20$</th>
<th>$E_p(t)$: $K_2=0.15$</th>
<th>$E_p(t)$: $K_2=0.10$</th>
<th>$E_p(t)$: $K_2=0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.493</td>
<td>0.743</td>
<td>0.793</td>
<td>0.843</td>
<td>0.893</td>
<td>0.943</td>
</tr>
<tr>
<td>2</td>
<td>0.975</td>
<td>1.475</td>
<td>1.575</td>
<td>1.675</td>
<td>1.775</td>
<td>1.875</td>
</tr>
<tr>
<td>3</td>
<td>1.451</td>
<td>2.201</td>
<td>2.351</td>
<td>2.501</td>
<td>2.651</td>
<td>2.801</td>
</tr>
<tr>
<td>4</td>
<td>1.923</td>
<td>2.923</td>
<td>3.123</td>
<td>3.323</td>
<td>3.523</td>
<td>3.723</td>
</tr>
<tr>
<td>6</td>
<td>2.856</td>
<td>4.356</td>
<td>4.656</td>
<td>4.956</td>
<td>5.256</td>
<td>5.556</td>
</tr>
<tr>
<td>7</td>
<td>3.319</td>
<td>5.069</td>
<td>5.419</td>
<td>5.769</td>
<td>6.119</td>
<td>6.469</td>
</tr>
</tbody>
</table>

8. Result and Conclusion

Table 1 and Fig. 2 provide information how availability of the complex repairable system changes with respect to time when the failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_1 = 0.05$, $\lambda_2 = 0.06$, $\lambda_3 = 0.01$, $\lambda_4 = 0.02$, $\lambda_5 = 0.03$, $\lambda_{cl} = 0.015$, $\lambda_w = 0.012$, $\lambda_c = 0.025$, availability of the system decreases and probability of failure increases, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predict the future behavior of complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Table 2 yields the mean-time-to-failure (MTTF) of the system with respect to variations in $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_5$, $\lambda_c$, $\lambda_w$, and $\lambda_{cl}$ respectively when other parameters
have been taken as constant. Fig. 3 shows the variations in MTTF corresponding to failure rates. Evidently, the MTTF decreases as failure rate increases. The MTTF corresponding to failure rate $\lambda_S$ increases which indicates that the failure $\lambda_S$ is more responsible for proper operation of the system.

When revenue cost per unit time $K_1$ is fixed at 1, service cost $K_2 = 0.5, 0.25, 0.20, 0.15, 0.10, 0.05$, profit has been calculated and results are demonstrated by graphs in figure 3. It can be observed that as service cost decreases, profit increases.

References