Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 8, Issue 2 (2015): 119-133

STOCHASTIC ANALYSIS OF N- UNIT PLANT WITH DIFFERENT TYPES OF FAILURE AND REPAIR POLICY USING COPULA DISTRIBUTION

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Received January 18, 2015
Modified December 14, 2015
Accepted December 22, 2015

Abstract

This paper deals with the reliability and profit analysis of an N-unit plant with two types of failure i.e. Partial failure and complete failure. The plant has N- units and at least k units are required to be in operational mode to fulfill the satisfactory needs of customer demand. Till the time k units are in good operational condition, i.e. at most (k-1) units fail, the plant can fulfill the needs of market demands and failure of more than k units it cannot satisfy customer requirements. Other types of failures like human failure and deliberate failure can also affect the functioning of plant. An important aspect of system failure, which is usually observed in industry, the strike of employees is also considered for study of mathematical model. The natural calamities such as earthquake, tsunami and sudden catching of fire that can fail the system completely, have also been considered in study of the model. The system has been studied by supplementary variable technique and various measures of reliability, such as availability, state transition probabilities, MTTF and profit function have been discussed. Some particular cases have also been discussed by taking different values of failure rates.

Key Words: Controller Failure, Deliberate Failure, Gumbel-Hougaard Family Copula, MTTF, Natural Calamity, Strike, Supplementary Variables.

1. Introduction

Reliability plays an important role in industrial automation. A high-level reliability is being demanded everywhere in industry and organizations. In this aspect, the various researchers have done a remarkable work in designing the systems for future demands. Redundancy is one of the techniques, which improves the reliability of system. The k-out- of-n system is an arrangement of unit in parallel configuration in such a way that the entire system performs the task until the k units are in good working condition. Only one operational unit of any industrial organization can not fulfill the necessity of market demand. In real life situations, it is observed that after some time the operational unit has to shut down for time to reduce the negative effect like heating, cooling and maintenance of oiling etc. Moreover, continuously running a single unit may have adverse effect and has a possibility of damaging some important part of system. Therefore, it is necessary to design the system, which consists of some unit in standby mode that can perform the task whenever it is required. Consecutive k-out- n systems have been studied by Kontoleon [8] and Ramamuthy [14] who proposed explicit formulas for- k-out-of-n systems. A consecutive-k-out-n: F-system consists of n ordered components along a line or circle such that the system fails if at least k consecutive components fail.

Many researchers have paid their attention to evaluate reliability and availability of repairable k-out of-n: good and k-out-of–n: fail systems. A, k- out –of –n system is found to have extensive applications in many fields of engineering and science, including power plants, airplane model with duet engine and industrial organizations model. Xiaolin Liang et al. [21] studied a consecutive k-out-of-n repairable system under the concept of k-component, or ordinary component such that the priority of repair assigned for k components. The authors [19] have studied the operational behavior of multi state- state k- out–of- n: type analysis of system by considering 2-out-of-3: G, system as a special case for computations. The study of Ibrahim Yusuf et al. [7], focused on the comparative study of 2- out – of – 3: systems for different situations under the concept of general repair. So many researchers including [1, 3, 13, 19, 20, 22] have studied the complex systems by considering the fact that the failed system can be repaired by general repair policy. They studied the reliability characteristic of complex systems under the fact that only one repair can be employed between two-transition states, but there are many situations where more than one repair is required between two adjacent transition states. When such possibility is observed, the system is studied using copula [12]. The authors [16] have also studied the complex systems having three units-super priority, priority and ordinary, under preemptive resume repairs policy. Alka Munjal et al. [2] have studied a complex repairable system composed of a 2- out- of -3: G; subsystems connected in parallel configuration. In modern competitive world, advance technology has introduced many electronic devices, which can improve the efficiency of system. The controller is one of such electronic devices, which is widely used in many electronic systems and equipments. In consequence to the study of complex systems with controller the researchers' including [17, 18] have discussed the reliability measures of a system, having two subsystems with controller, human and deliberate failure. Various researchers have extensively studied the standby redundant complex systems. In reference to the study of standby redundant complex systems, Singh et al. [11] have studied availability of standby complex system under waiting repair and human failure using Gumbel-Hougaard family copula distribution. Furthermore, number of researchers studied reliability measures and tried to obtain best structure of field to fulfill the challenges of industry requirements, but still more attention is required in this field.

Considering necessity and requirements of future prospects the study of such type of systems, the authors have described a mathematical model of N- unit plant, which works under k-out- of- n: good policy. Initially in state S_0 the system is in perfect state, where all units are in good working condition. When the system starts functioning, due to failure of some (k - i), i= 1, 2, 3..., units in the system, it approaches to the degraded state S_1. Further failure of more than k units, the system approaches to completely failed state S_2. Human and deliberate failure brings the system in to complete failed states S_3 and S_6 respectively. An automatic controller, which controls the functioning of the plant should be considered as important phase of system failure. Controller failure brings the system into complete failed state S_8. In real life experience, strike of employees is a routine phenomenon seen in industry, which affects the functioning of plant/ organization. The state S_4 and S_5 represents when employees strike appears and then the system has to wait for further operation. The natural calamities such as earthquake, tsunami etc which may completely damage the system are represented by the state S_6. Whenever the system comes in degraded mode, general repair may be employed but in completely failed states of the system repair can be accomplished by Gumbel- Hougaard family copula distribution. The system is analyzed by supplementary variable technique, various measures of reliability have also been discussed and some particular cases are taken to highlight the results. Results and conclusions are described by Tables and graphs.

The entire paper has been divided into following sections; Section I of paper is introduction, which consists the related work done by previous researchers and need of study is highlighted. Section II of paper is state description for clearly understanding the description of various states. Section III consists of the basic assumptions for elaborated model. In section, IV and V are the state transition diagram and notations that have been used for mathematical modeling and solution respectively. In section VI, paper covers mathematical modeling and solution of formulated model. The VII section of the paper is an analytical part in which the various measures of reliability like availability, MTTF and profit analysis have been evaluated for different values of parameters. The last VII section of paper includes conclusion part for discussion of results and their explanation for future prospects.

2. State Description

State	State Description
S_0	All units of system are good working condition and system is in perfect state.
S_1	The state S_1 represents that, at least k unit of system are in good working condition.
S_2	In S_2 state the system has fail due to failure of (k +i) units of system. i = 1, 2, 3………..
S_3	The state S_3 represents appearance of human failure due to which, the system approaches to complete failure mode.
S_4	In state S_4, the system is in degraded state due to sudden strike of employees.
S_5	State S_5 represents the state where system is suppose to wait for further functioning due to strike.
S_6	In state S_6 the system has been failed due to deliberate failure by operator.
S_7	The system fails completely due to failure of natural calamity that appears in state S_7.
S_8	System fails completely due to controller failure, which may be seen in state S_8.

3. Assumptions

The following assumptions are taken throughout the discussion of model.

(1) Initially the system is in perfect state S_0 and all units are in good working condition.

(2) The system works successfully until at least k- units of it is in good working condition.

(3) System fails if more than k units fail.

(4) Only one change is allowed at a time in the transitions.

(5) The strike of employees stop functioning of system and due to strike the system waits for restarts functioning.

(6) Deliberate failure, human failure as well as natural calamity completely fail the system.

(7) Partially failed states have been repaired by employing general time distribution.

(8) Deliberate failure / human failure / natural calamity and controller failure system need fast repairing and hence they are repaired by using (Gumbel-Hougaard) family copula distribution.

(9) Repaired system works like a new and repair did not damage anything.

4. State Transition Diagram of Model

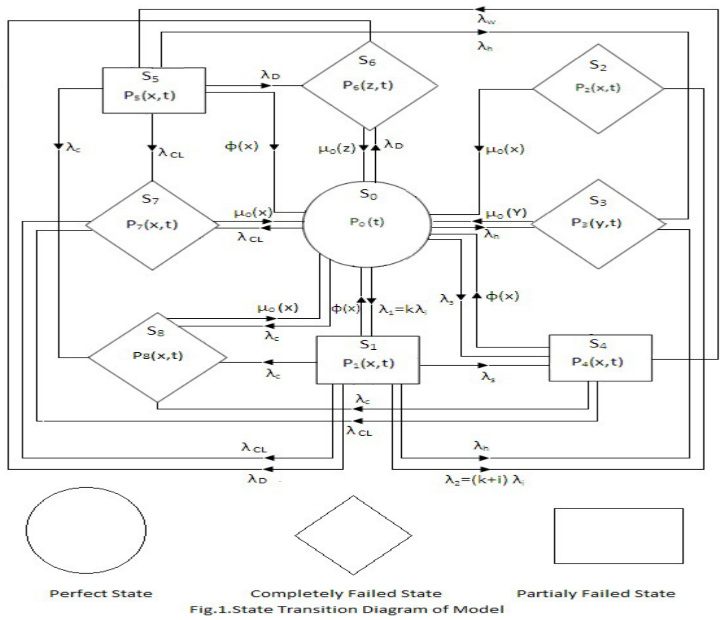

Fig.1: State Transition Diagram of Model

5. Notations

λ_1 / λ_2	Failure rates of system such that at most k unit /more than k units failed during operational mode.
$\lambda_D / \lambda_{CL} / \lambda_C / \lambda_S / \lambda_h$	Failure rate of deliberate failure/ failure due to natural calamities/ controller failure/ failure due to strike/ human failure.
$\phi(x)$	General repair rate of system for degraded state i.e. state S_1 and S_5
$P_i(t)$	Represents state transition probabilities of respective states of the system.
$\overline{P}(s)$	Laplace transform of state transition probabilities P(t).
$P_i(x, t)$	State transition probability that the system is in state S_i , i=1,.... 8. $P_i(x, t)$ represents that the system in state i. The system is in repair with repair variables x, t.
$\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}$	The expression for joint probability distribution (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula for given ϕ(x)= general repair, e^x = exponential repair rate and coupling both gives $\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}$ =Ø(x) e^x
$E_p(t)$	Expected profit in interval [0,t)

6. Formulation of Mathematical Model

By probability considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model.

$$\left[\frac{\partial}{\partial t} + \lambda_1 + \lambda_{CL} + \lambda_C + \lambda_h + \lambda_D + \lambda_S\right] P_0(t) = \int_0^\infty \phi(x) P_1(x,t) dx + \int_0^\infty \exp[y^\theta + \{\log\phi(y)\}^\theta]^{1/\theta} P_8(y,t) dy$$
$$+ \int_0^\infty \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta} P_7(x,t) dx + \int_0^\infty \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta} P_2(x,t) dx + \int_0^\infty \phi(x) P_4(x,t) dx$$
$$+ \int_0^\infty \exp[z^\theta + \{\log\phi(z)\}^\theta]^{1/\theta} P_6(z,t) dz + \int_0^\infty \phi(x) P_5(x,t) dx + \int_0^\infty \exp[y^\theta + \{\log\phi(y)\}^\theta]^{1/\theta} P_3(y,t) dy \quad (6.1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_C + \lambda_{CL} + \lambda_D + \lambda_h + \lambda_S + \phi(x)\right] P_1(x,t) = 0 \quad (6.2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}\right]P_2(x,t)=0 \tag{6.3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\exp[y^{\theta}+\{\log\phi(y)\}^{\theta}]^{1/\theta}\right]P_3(y,t)=0 \tag{6.4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_W+\lambda_C+\lambda_{CL}+\phi(x)\right]P_4(x,t)=0 \tag{6.5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{CL}+\lambda_C+\lambda_D+\lambda_h+\phi(x)\right]P_5(x,t)=0 \tag{6.6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\exp[z^{\theta}+\{\log\phi(z)\}^{\theta}]^{1/\theta}\right]P_6(z,t)=0 \tag{6.7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}\right]P_7(x,t)=0 \tag{6.8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\exp[y^{\theta}+\{\log\phi(y)\}^{\theta}]^{1/\theta}\right]P_8(y,t)=0 \tag{6.9}$$

Boundary Conditions

$$P_1(0,t)=\lambda_1 P_0(t) \tag{6.10}$$

$$P_2(0,t)=\lambda_1\lambda_2 P_0(t) \tag{6.11}$$

$$P_3(0,t)=\lambda_h P_0(t)+\lambda_h P_5(0,t)+\lambda_h P_1(0,t) \tag{6.12}$$

$$P_4(t)=\lambda_S P_0(t)+\lambda_S P_1(0,t) \tag{6.13}$$

$$P_5(0,t)=\lambda_w P_4(0,t)=\lambda_w\lambda_S P_0(t) \tag{6.14}$$

$$P_6(0,t)=\lambda_D P_0(t)+\lambda_D P_1(0,t)+\lambda_D P_5(0,t) \tag{6.15}$$

$$P_7(0,t)=\lambda_{CL} P_0(t)+\lambda_{CL} P_1(0,t)+\lambda_{CL} P_5(0,t)+\lambda_{CL} P_4(0,t) \tag{6.16}$$

$$P_8(0,t)=\lambda_C P_0(t)+\lambda_C P_1(0,t)+\lambda_C P_4(0,t)+\lambda_C P_5(0,t) \tag{6.17}$$

Initial Conditions

$P_0(0)=1$ and other state probabilities are zero at t = 0 (6.18)

Solution of the Model

Taking Laplace transformation of equations (6.1)-(6.17) and using equation (6.18), we obtain.

$$\left(s+\lambda_1+\lambda_{CL}+\lambda_C+\lambda_D+\lambda_h+\lambda_S\right)\overline{P}_0(s)=[1+\int_0^\infty \overline{P}_1(x,s)\phi(x)dx+\int_0^\infty \overline{P}_8(y,s)\exp[y^\theta+\{\log\phi(y)\}^\theta]^{1/\theta}dy$$
$$+\int_0^\infty \overline{P}_7(x,s)\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}dx+\int_0^\infty \overline{P}_3(y,s)\exp[y^\theta+\{\log\phi(y)\}^\theta]^{1/\theta}dy+\int_0^\infty \overline{P}_5(x,s)\phi(x)dx+$$
$$\int_0^\infty \overline{P}_4(x,s)\phi(x)dx+\int_0^\infty \overline{P}_2(x,s)\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}dx+\int_0^\infty \overline{P}_6(z,s)\exp[z^\theta+\{\log\phi(z)\}^\theta]^{1/\theta}dz] \tag{6.19}$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_2+\lambda_C+\lambda_D+\lambda_S+\lambda_h+\lambda_{CL}+\phi(x)\right]\overline{P}_1(x,s)=0 \tag{6.20}$$

$$\left[s+\frac{\partial}{\partial x}+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}\right]\overline{P}_2(x,s)=0 \tag{6.21}$$

$$\left[s+\frac{\partial}{\partial y}+\exp[y^\theta+\{\log\phi(y)\}^\theta]^{1/\theta}\right]\overline{P}_3(y,s)=0 \tag{6.22}$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_w+\lambda_{CL}+\lambda_C+\phi(x)\right]\overline{P}_4(x,s)=0 \tag{6.23}$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_{CL}+\lambda_C+\lambda_D+\lambda_h+\phi(x)\right]\overline{P}_5(x,s)=0 \tag{6.24}$$

$$\left[s+\frac{\partial}{\partial z}+\exp[z^\theta+\{\log\phi(z)\}^\theta]^{1/\theta}\right]\overline{P}_6(z,s)=0 \tag{6.25}$$

$$\left[s+\frac{\partial}{\partial x}+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}\right]\overline{P}_7(x,s)=0 \tag{6.26}$$

$$\left[s+\frac{\partial}{\partial y}+\exp[y^\theta+\{\log\phi(y)\}^\theta]^{1/\theta}\right]\overline{P}_8(y,s)=0 \tag{6.27}$$

$$\overline{P}_1(0,s)=\lambda_1\overline{P}_0(s) \tag{6.28}$$

$$\overline{P}_2(0,s)=\lambda_1\lambda_2\overline{P}_0(s) \tag{6.29}$$

$$\overline{P}_3(0,s)=\lambda_h\overline{P}_0(s)+\lambda_h\overline{P}_5(0,s)+\lambda_h\overline{P}_1(0,s) \tag{6.30}$$

$$\overline{P}_4(0,s)=\lambda_S(\overline{P}_0(s)+\overline{P}_1(0,s)) \tag{6.31}$$

$$\overline{P}_5(0,s)=\lambda_W\lambda_S\overline{P}_0(s) \tag{6.32}$$

$$\overline{P}_6(0,s)=\lambda_D(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_5(0,s)) \tag{6.33}$$

$$\overline{P}_7(0,s)=\lambda_{CL}(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_5(0,s)+\overline{P}_4(0,s)) \tag{6.34}$$

$$\overline{P}_8(0,s)=\lambda_C(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_5(0,s)+\overline{P}_4(0,s)) \tag{6.35}$$

Solving (6.20) -(6.27) with the help of (6.28) -(6.35), one may get

$$\overline{P}_0(s) = \frac{1}{D(s)} \tag{6.36}$$

$$\overline{P}_1(s) = \frac{\lambda_1}{D(s)} \frac{(1 - S_\phi(s + \lambda_2 + \lambda_C + \lambda_{CL} + \lambda_h + \lambda_S + \lambda_D))}{(s + \lambda_2 + \lambda_C + \lambda_{CL} + \lambda_D + \lambda_h + \lambda_S)} \tag{6.37}$$

$$\overline{P}_2(s) = \frac{\lambda_1 \lambda_2}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{s} \tag{6.38}$$

$$\overline{P}_3(s) = \frac{\lambda_h(1 + \lambda_1 + \lambda_w \lambda_S)}{D(s)} \left\{ \frac{(1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{s} \right\} \tag{6.39}$$

$$\overline{P}_4(s) = \frac{\lambda_S(1 + \lambda_1)}{D(s)} \left\{ \frac{(1 - S_\phi(s + \lambda_w + \lambda_C + \lambda_{CL})}{(s + \lambda_w + \lambda_C + \lambda_{CL})} \right\} \tag{6.40}$$

$$\overline{P}_5(s) = \frac{\lambda_w \lambda_S}{D(s)} \left\{ \frac{(1 - S_\phi(s + \lambda_h + \lambda_C + \lambda_{CL} + \lambda_D)}{(s + \lambda_h + \lambda_C + \lambda_{CL} + \lambda_D)} \right\} \tag{6.41}$$

$$\overline{P}_6(s) = \frac{\lambda_D(1 + \lambda_1 + \lambda_w \lambda_S)}{D(s)} \left\{ \frac{(1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{s} \right\} \tag{6.42}$$

$$\overline{P}_7(s) = \frac{\lambda_{CL}(1 + \lambda_1 + \lambda_w \lambda_S + \lambda_S \lambda_1 + \lambda_S)}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{s} \tag{6.43}$$

$$\overline{P}_8(s) = \frac{\lambda_C(1 + \lambda_1 + \lambda_S \lambda_1 + \lambda_S + \lambda_w \lambda_S)}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{s} \tag{6.44}$$

$$D(s) = \left\{ \begin{array}{l} s + \lambda_1 + \lambda_C + \lambda_{CL} + \lambda_D + \lambda_h + \lambda_S - [\lambda_1 \bar{S}_\phi(s + \lambda_2 + \lambda_{CL} + \lambda_C + \lambda_D + \lambda_h + \lambda_S) + \\ \lambda_{CL}(1 + \lambda_1 + \lambda_S \lambda_w + \lambda_S + \lambda_S \lambda_1) S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) + \\ (\lambda_C(1 + \lambda_1 + \lambda_S + \lambda_S \lambda_1 + \lambda_S \lambda_w) S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) + \lambda_D(1 + \lambda_1 + \lambda_S \lambda_w) S_{\exp[z^\theta + \{\log\phi(z)\}^\theta]^{1/\theta}}(s) \\ + \lambda_w \lambda_S S_\phi(s + \lambda_h + \lambda_C + \lambda_{CL} + \lambda_D) + \lambda_h(1 + \lambda_w \lambda_S + \lambda_1) S_{\exp[y^\theta + \{\log\phi(y)\}^\theta]^{1/\theta}}(s) + \\ (\lambda_S(1 + \lambda_1) S_\phi(s + \lambda_C + \lambda_W + \lambda_{CL}) + \lambda_1 \lambda_2 S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s)] \end{array} \right\} \tag{6.45}$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_4(s) + \overline{P}_5(s)$$

$$=\frac{1}{D(s)}\left[\begin{array}{l}1+\lambda_1\left\{\frac{(1-S_\varphi(s+\lambda_2+\lambda_C+\lambda_S+\lambda_h+\lambda_{CL}+\lambda_D))}{(s+\lambda_2+\lambda_C+\lambda_{CL}+\lambda_D+\lambda_S+\lambda_h)}\right\}+\lambda_S(1+\lambda_1)\left\{\frac{(1-S_\varphi(s+\lambda_w+\lambda_C+\lambda_{CL})}{(s+\lambda_w+\lambda_C+\lambda_{CL})}\right\}\\+\lambda_w\lambda_S\left\{\frac{(1-S_\varphi(s+\lambda_h+\lambda_C+\lambda_{CL}+\lambda_D)}{(s+\lambda_h+\lambda_C+\lambda_{CL}+\lambda_D)}\right\}\end{array}\right] \tag{6.46}$$

$$\overline{P}_{failed}(s)=\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_6(s)+\overline{P}_7(s)+\overline{P}_8(s) \tag{6.47}$$

7. Particular Cases

7 A: Availability Analysis

When repair follows exponential time distribution;
Using the formulas

$$\overline{S}_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s)=\frac{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}},\ \overline{S}_\phi(s)=\frac{\phi}{s+\phi}$$

in equation (6.46) and setting the values of different parameters as λ_1=0.05, λ_2=0.06, λ_C = 0.025, λ_D = 0.02, λ_{CL} = 0.015, λ_w = 0.012, λ_h = 0.01, λ_S = 0.03, ϕ = 1, θ = 1, x = 1 ,y=1,z=1 then taking inverse Laplace transform, one can obtain expression for availability of system as;

$$P_{up}(t)=-0.005017342e^{(-2.79348t)}+0.0008933e^{(-1.20836t)}+0.00012558e^{(-1.070313t)}$$
$$+0.0226998e^{(-1.02314t)}+0.981298e^{(-0.00299339t)} \tag{7.1}$$

For, t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.... One may get different values of P_{up} (t) as shown inTable1 and corresponding figure 2.

Time (t)	Availability
0	1.000
1	0.987
2	0.978
3	0.974
4	0.970
5	0.967
6	0.964
7	0.961
8	0.958
9	0.955

Table 1.Variation of Availability with respect to time

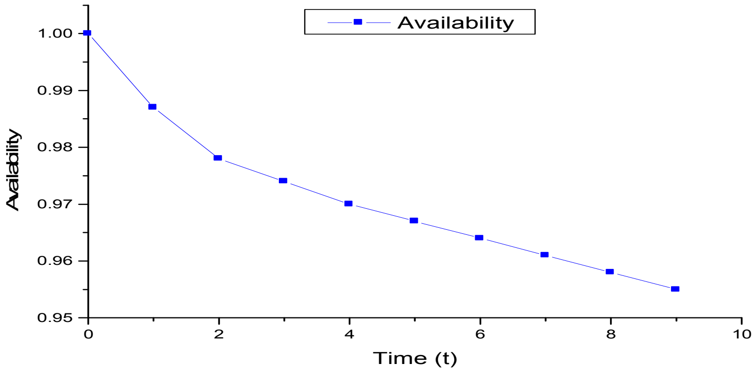

Fig. 2: Time v/s Availability

7 B: Mean Time to Failure (MTTF)

Setting

$$\bar{S}_{\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}}(s)=\frac{\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}}{s+\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}},\quad \bar{S}_{\phi}(s)=\frac{\phi}{s+\phi}$$

and taking all repairs to zero in equation (6.46). Taking limit, as s tends to zero one can obtain the MTTF as:

$$M.T.T.F=\frac{1}{(\lambda_1+\lambda_C+\lambda_{CL}+\lambda_D+\lambda_h+\lambda_S)}\left[1+\frac{\lambda_1}{(\lambda_2+\lambda_C+\lambda_{CL}+\lambda_D+\lambda_S+\lambda_h)}+\frac{\lambda_S(1+\lambda_1)}{(\lambda_w+\lambda_C+\lambda_{CL})}+\frac{\lambda_w\lambda_S}{(\lambda_h+\lambda_C+\lambda_{CL}+\lambda_D)}\right] \tag{7.2.1}$$

Setting, λ_2=0.06, λ_D=0.02, λ_h=0.01, λ_S=0.03, λ_{CL}=0.015, λ_C=0.025, λ_w=0.012, in above expression (7.2.1) and then varying λ_1 as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 respectively, one can obtain the values of MTTF with respect to failure rate λ_1. The second column of Table 3 demonstrates the variation of MTTF with respect to failure rate λ_1.

Setting , λ_1=0.05, λ_D=0.02, λ_h=0.01, λ_S=0.03, λ_{CL}=0.015, λ_C=0.025, λ_w=0.012, and varying λ_2 as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) we can get values of MTTF corresponding to failure rate λ_2. Third column of Table 3 represents the values of failure rate corresponding to failure rate λ_2.

Setting λ_2=0.06, λ_1=0.05, λ_h=0.01, λ_S=0.03, λ_{CL}=0.015, λ_C=0.025, λ_w=0.012, and varying λ_D as: 0.01, 0.02, 0.03, 0.04, 0.005, 0.06, 0.07, 0.08, 0.09 in (7.2.1) we can obtain MTTF respect to failure rate λ_D. Column 4 of Table 3 shows variation of MTTF with respect to λ_D.

Setting λ_2=0.06, λ_D=0.02, λ_1=0.05, λ_S=0.03, λ_{CL}=0.015, λ_C=0.025, λ_w=0.012, and varying λ_h as: 0.01, 0.002, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one can obtain MTTF with respect to failure rate $\lambda_{h.}$ Column 5 of Table 3 reveals variation of MTTF with respect to λ_h.

Setting λ_2=0.06, λ_D=0.02, λ_1=0.05, λ_h=0.01, λ_{CL}=0.015, λ_C=0.025, λ_w=0.012, and then varying λ_S as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one obtain MTTF respect to failure rate λ_S. Column 6 of Table 3 shows variation of MTTF with respect to λ_D.

Setting λ_2=0.06, λ_D=0.02, λ_1=0.05, λ_S=0.03, λ_h=0.015, λ_C=0.025, λ_w=0.012, and then varying λ_{CL} as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one obtain MTTF with respect to λ_{CL}. The computed values of MTTF are shown in column7 of Table 3.

Setting λ_2=0.06, λ_D=0.02, λ_1=0.05, λ_S=0.03, λ_{CL}=0.015, λ_h=0.01, λ_w=0.012, and then varying λ_C as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one can obtain MTTF with respect to λ_C shown in column8 of the Table 3.

Setting λ_2=0.06, λ_D=0.02, λ_1=0.05, λ_S=0.03, λ_{CL}=0.015, λ_C=0.025, λ_h=0.01, and varying λ_w as: 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (7.2.1) one can get MTTF values for the failure rate λ_w in column9 in Table 3 given as under.

Failure rate	MTTF λ_1	MTTF λ_2	MTTF λ_D	MTTF λ_h	MTTF λ_S	MTTF λ_{CL}	MTTF λ_C	MTTF λ_w
0.01	15.0030	13.7697	13.8936	12.8227	12.0060	13.7816	16.3165	12.9786
0.02	14.3217	13.5172	12.8227	11.9024	12.4329	12.0030	13.7816	12.3071
0.03	13.7452	13.3035	11.9024	11.1032	12.8227	10.6673	12.0030	11.8357
0.04	13.2510	13.1204	11.1032	10.4029	13.1792	9.6180	10.6673	11.4893
0.05	12.8227	12.9616	10.4029	9.7844	13.5057	8.7671	9.6180	11.2262
0.06	12.4480	12.8227	9.7844	9.2343	13.8055	8.0604	8.7671	11.0214
0.07	12.1174	12.7002	9.2343	8.7420	14.0814	7.4626	8.0604	10.8591
0.08	11.8234	12.5913	8.7420	8.2988	14.3360	6.9494	7.4626	10.7286
0.09	11.5605	12.4938	8.2988	7.8978	14.5715	6.5035	6.9494	10.6225
0.10	11.3238	12.4061	7.8978	7.5334	14.7898	6.1120	6.5035	10.5357

Table 2. Computation of MTTF for different values of failure rates.

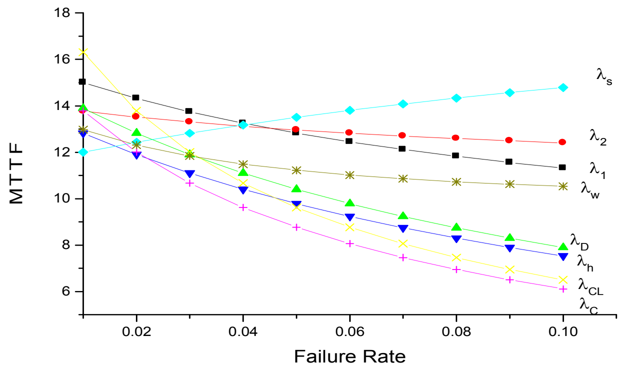

Fig. 3: Variation of MTTF with respect to failure rates

7 C: Cost Analysis

Let the failure rates of system be λ_1=0.05, λ_2=0.06, λ_C = 0.025, λ_D = 0.02, λ_{CL} = 0.015, λ_w =0.012, λ_h =0.01, λ_S =0.03, mean time to repair of be $\phi(x)$=1, x=1, θ=1, and Setting,

$$\overline{S}_{\exp[x^\theta+\{\log \phi(x)\}^\theta]^{1/\theta}}(s)=\frac{\exp[x^\theta+\{\log \phi_A(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log \phi_A(x)\}^\theta]^{1/\theta}},\ \overline{S}_\phi(s)=\frac{\phi}{s+\phi}$$ in

equation (6.46) and taking inverse Laplace transform, one can obtain (7.3.1).

Let the service facility be always available, then expected profit during the interval [0, t) is

$$E_p(t)=K_1\int_0^t P_{up}(t)dt-K_2t$$

Where K_1 and K_2 are revenue service cost per unit time. Hence

$$E_p(t)=K_1(0.001796e^{(-2.793487t)}-0.000739e^{(-1.208364t)}-0.000117e^{(-1.070313t)}$$
$$-0.022186e^{(-1.023140t)}-327.755101e^{(-0.0012993t)}+327.77634)-K_2t \tag{7.3.1}$$

Setting K_1= 1 and K_2= 0.5, 0.25, 20, 0.15, 0.10, and 05 respectively and varying t =0,1, 2, 3, 4, 5, 6, 7, 8, 9,......, one get Table 4, which give the expected profit with variation of time t.

Time(t)	$E_p(t)$; K_2=0.5	$E_p(t)$; K_2=0.25	$E_p(t)$; K_2=0.20	$E_p(t)$; K_2=0.15	$E_p(t)$; K_2=0.10	$E_p(t)$; K_2=0.05
0	0	0	0	0	0	0
1	0.493	0.743	0.793	0.843	0.893	0.943
2	0.975	1.475	1.575	1.675	1.775	1.875
3	1.451	2.201	2.351	2.501	2.651	2.801
4	1.923	2.923	3.123	3.323	3.523	3.723
5	2.391	3.641	3.891	4.141	4.391	4.641
6	2.856	4.356	4.656	4.956	5.256	5.556
7	3.319	5.069	5.419	5.769	6.119	6.469
8	3.778	5.778	6.178	6.578	6.978	7.378
9	4.235	6.485	6.935	7.385	7.835	8.285

Table 3. Computation of Expected profit

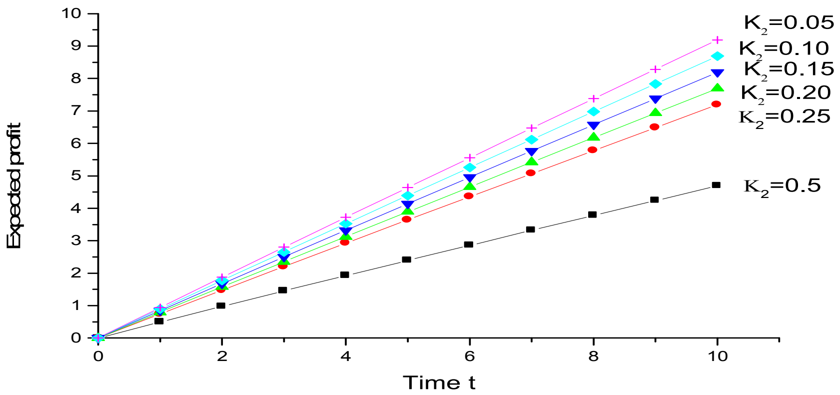

Fig 4. for Expected profit with variation of time

8. Result and Conclusion

Table 1 and Fig. 2 provide information how availability of the complex repairable system changes with respect to time when the failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_1 = 0.05$, $\lambda_2 = 0.06$, $\lambda_h = 0.01$, $\lambda_D = 0.02$, $\lambda_S = 0.03$, $\lambda_{CL} = 0.015$, $\lambda_w = 0.012$, $\lambda_C = 0.025$, availability of the system decreases and probability of failure increases, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predict the future behavior of complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Table 2 yields the mean-time-to-failure (MTTF) of the system with respect to variations in λ_1, λ_2, λ_D, λ_h, λ_S , λ_C, λ_w, and λ_{CL} respectively when other parameters have been taken as constant. Fig. 3 shows the variations in MTTF corresponding to failure rates. Evidently, the MTTF decreases as failure rate increases. The MTTF corresponding to failure rate λ_S increases which indicates that the failure λ_S is more responsible for proper operation of the system.

When revenue cost per unit time $K_{1\ is}$ fixed at 1, service cost $K_2 = 0.5, 0.25, 0.20, 0.15, 0.10, 0.05$, profit has been calculated and results are demonstrated by graphs in figure 3. It can be observed that as service cost decreases, profit increases.

References

1. A. K. Govil. (1974). Operational behaviour of a complex system having shelf-life of the components under preemptive-resume repair discipline, Microelectronics & Reliab, 13(2), p. 97-101.
2. Alka Munjal and S. B. Singh. (2014). Reliability analysis of a complex repairable system composed of a 2- out-of-3: G: sub system and a series subsystem connected in parallel, Journal of Reliability and Statistical studies, vol. 7 (special issue), p.19- 39.
3. K. K, Aggarwal, J. S. Gupta, and K. B. Mishra. (1975). A simple method for reliability evaluation of communication system, IEEE Transaction on Communications, 23(5), p.563-565.
4. Chiwa Musa Dalah, V. V. Singh (2014). Study of reliability measures of a two unit standby system under the concept of switch failure using copula distribution, American Journal of Computational and Applied Mathematics, 4(4), p.118-129.
5. D. K. Rawal, Mangey Ram and V. V. Singh. (2014). Modeling and availability analysis of internet data center with various maintenance policies, International Journal of Engineering, Transactions A: Basics, 27(4), p. 599-608.
6. Heinrich Gennheimer. (2002). Model Risk in Copula Based Default Pricing Models, Working Paper Series, Working Paper No. 19, Swiss Banking Institute, University of Zurich and NCCR FINRISK.
7. Ibrahim. Yusuf, N. Hussaini. (2014). A comparative analysis of three unit redundant systems with three types of failures, Arab J Sci Eng, 39(4), p. 3337-3349.
8. J. M. Kontoleon. (1980). Reliability determination of an r-successive–out-of n: F system, IEEE. Trans. Reilab, 29(5), p. 437-447.
9. Sun-Wah. Kui, Mc Allister D.F. (1988). Reliability optimization of computer communication network, IEEE Transaction on reliability, 37(5), p.475-483.
10. Lirong Cui, Haijun. Li. (2007). Analytical method for reliability and MTTF assessment of coherent systems with dependent components, Reliability Engineering & System Safety, 92(3), p.300-307.
11. Mangey Ram, S. B. Singh and V. V. Singh. (2013). Stochastic analysis of a standby complex system with waiting repair strategy, IEEE Transactions on System, Man, and Cybernetics Part A: System and humans, 43(3), p. 698-707.
12. R.B. Nelsen. (2006). An Introduction to Copulas (2nd edn.) New York, Springer.

13. Ram Niwas and M. S. Kadyan. (2015). Reliability modeling of a maintained system with warranty and degradation, Journal of Reliability and Statistical studies, 8(1), p. 63-75.
14. K.G. Ramamurthi. (1997). The reliability function of consecutive k-out-of-n systems for the i. i. d case, Sankhya; The Indian Journal of Statistics Series B, 59(2), p. 209-228.
15. S. B. Singh, P. P. Gupta and C. K. Goel. (2001). Analytical study of the complex stand by redundant system involving the concepts of multi-failure, human failure under head of line repair discipline, Bulletin of Pure and Applied Sciences, 20 E (2), p. 345-351.
16. V. V. Singh, S. B. Singh, Mangeyram and C. K. Goel. (2010). Availability analysis of a system having three units super priority, priority and ordinary under preemptive resume repair policy, International journal of reliability and applications, 11(1), p. 41-53.
17. V. V. Singh, S. B. Singh, M. Ram and C. K. Goel. (2012). Availability, MTTF and cost analysis of a system having two units in series configuration with controller, International Journal of System Assurance and Management, 4(4), p.341-352.
18. V. V. Singh and D. K Rawal. (2011). Availability analysis of a system having two units in series configuration with controller and human failure under different repair policies, Inter. J. of Scient. Eng. Research, 2 (10), p. 1-9.
19. V. V. Singh and Mangey Ram. (2014). Multi-state k-out-of-n type system analysis, Mathematics in Engineering, Science and Aerospace, 5(3), p.281-292.
20. G. J. Wang, Y. L. Zhang. (2007). An optimal replacement policy for a two-component series system assuming geometric process repair, Computers & Mathematics with Applications, 54(2), p. 192–202.
21. Xiaolin Liang, Yinfeng and Zehui. Li. (2010). Exact Reliability formula for consecutive k-out of –n Repairable systems, IEEE Transaction on Reliability, 59(2), p.313-318.
22. Zhang. T, Horigome. M. (2001). Availability and reliability of system with dependent components and time-varying failure and repair rates, IEEE Trans Reliab 50(2), p.151–158.