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A COMPARATIVE STUDY UNDER PROGRESSIVELY FIRST FAILURE CENSORED RAYLEIGH DATA

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Abstract

A comparative study presented in this article for two different asymmetric loss functions is based on simulation. Two-parameter Rayleigh model is considered here as the underline model for evaluating the properties of Bayes estimators under progressive first failure censored data. Known and unknown both cases of location parameter are considered here for Bayes estimation of scale parameter.

Key Words: Bayes Estimator, Progressive First Failure Censoring Scheme, ISELF, LLF.

1. Introduction

The probability density function and distribution function of two - parameter Rayleigh distribution are given as

$$f\,(x;\theta,\sigma)=\frac{x-\sigma}{\theta^2}\exp\left(-\frac{(x-\sigma)^2}{2\theta^2}\right)\;;\,x>\sigma>0,\theta>0\,. \tag{1.1}$$

and

$$F\,(x;\theta,\sigma)=1-\exp\left(-\frac{(x-\sigma)^2}{2\theta^2}\right)\;;\,x>\sigma>0,\theta>0\,. \tag{1.2}$$

Here, the parameter θ is known as scale parameter and parameter σ is called as location parameter (Bain and Engelhardt (1992)). The considered model is useful in life testing experiments, in which age with time as its failure rate is a linear function of time. The present distribution also plays an important role in communication engineering and electro vacuum devices.

The Rayleigh distribution is often used in physics related fields to model processes such as sound and light radiation, wave heights, and wind speed, as well as in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year at wind turbine sites and daily average wind speed.

In the present paper, our focus is on presenting a comparative study on Bayes estimation under two different asymmetric loss functions based on Progressive first failure censored Rayleigh data. Known and unknown both cases of location parameter are considered here for Bayes estimation of scale parameter. For evaluation of performances of the proposed procedures, a simulation study has been carried out.

A good deal of literature is available on Rayleigh model under different criteria. A few of them are Sinha (1990), Fernandez (2000), Raqab and Madi (2002), Ali – Mousa and Al - Sagheer (2005), Wu et al. (2006), Abd - Elfattah et al. (2006), Kim and Han (2009) and Prakash and Prasad (2010).

Soliman et al. (2005) presents some estimators for finite mixture of Rayleigh model based on progressively censored data. Dey and Maiti (2012) have presented Bayes estimation for Rayleigh parameter under extended Jeffrey's prior. Bayes estimation based on Rayleigh progressive Type - II censored data with binomial removals was discussed by Azimi and Yaghmaei (2013). Some Bayesian analysis under Rayleigh model is also discussed recently by Ahmed et al. (2013).

2. The progressive first failure censoring

In many industrial experiments involving lifetimes of machines or units, experiments have to be terminated early and the number of failures must be limited for various reasons. The planning of experiments with aim of reducing total duration of experiment or the number of failures leads naturally to Type - I and Type - II censoring schemes.

If an experimenter desires to remove surviving units at points other than the final termination point of the life test, these two traditional censoring schemes will not be of use to the experimenter. The allowance of removing surviving units from the test before the final termination point is desirable, as in the case of studies of wear, in which the study of the actual aging process requires units to be fully disassembled at different stages of the experiment. In addition, when a compromise between the reduced time of experimentation and the observation of at least some extreme lifetimes is sought, such an allowance is also desirable. These reasons lead us into the area of progressive censoring.

It is well known that one of the primary goals of progressive censoring is to save some live units for other tests, which is particularly useful when units being tested are very expensive. Johnson (1964) described a life test in which the experimenter might decide to group the test units into several sets, each as an assembly of test units, and then run all the test units simultaneously until occurrence the first failure in each group. Such a censoring scheme is called first failure censoring. The first failure censoring scheme is terminated when first failure in each set is observed. If an experimenter desires to remove some sets of test units before observing first failures in these sets this life test plan is called a Progressive first failure censoring scheme which is recently introduced by Wu and Kus (2009).

The Progressive first failure censoring scheme is described as follows:

Suppose that n independent groups with k items within each group are put in a life test, R_1 groups and the group in which first failure is observed are randomly removed from the test as soon as first failure (say $X^R_{1:m:n:k}$) has occurred, R_2 groups and the group in which the first failure is observed are randomly removed from the test when second failure (say $X^R_{2:m:n:k}$) has occurred, and finally $R_m\,(m \le n)$ groups and the group in which the first failure is observed are randomly removed from the test as soon as the m^{th} failure (say $X^R_{m:m:n:k}$) has occurred. Here $X^R_{1:m:n:k} < X^R_{2:m:n:k} < \ldots < X^R_{m:m:n:k}$ are known as Progressive first failure censored order statistics with progressive censoring scheme $R_1, R_2, \ldots, R_m; (m \le n)$. Here, $R_i;\ \forall\, i = 1, 2, \ldots m \le n$ all are predefined integers follows the relation

$$\sum_{j=1}^{m} R_j + m = n .$$

If failure times of $n * k$ items originally in the test are from model (1.1), then joint probability density function for order statistics $X^R_{1:m:n:k} < X^R_{2:m:n:k} < \ldots < X^R_{m:m:n:k}$ is defined as

$$f_{X_{1:m:n:k}, X_{2:m:n:k}, \ldots, X_{m:m:n:k}}\left(\theta,\sigma|\underline{x}\right) = C_m k \prod_{i=1}^{m} f\left(x_{(i)};\theta,\sigma\right)\left(1 - F\left(x_{(i)};\theta,\sigma\right)\right)^{k(R_i+1)-1} \tag{2.1}$$

where $f(\cdot)$ and $F(\cdot)$ are given respectively by (1.1) and (1.2) and C_m is a progressive normalizing constant defined as $C_m = n(n - R_1 - 1)(n - R_1 - R_2 - 2)\ldots(n + 1 - \sum_{j=1}^{m-1} R_j - m)$ and Progressive first failure censored sample is denoted by $\underline{x} \equiv \left(x_{(1)}, x_{(2)}, \ldots, x_{(m)}\right)$.

Subtitling (1.1) and (1.2) in (2.1), the joint probability density function is obtained as:

$$f_{X_{1:m:n:k}, X_{2:m:n:k}, \ldots, X_{m:m:n:k}}\left(\theta,\sigma|\underline{x}\right) = C_m H_m(\underline{x},\sigma)\, \theta^{-2m} \exp\left(-\frac{T_{pf}(\underline{x},\sigma)}{2\theta^2}\right) \tag{2.2}$$

where $H_m(\underline{x},\sigma) = \prod_{i=1}^{m}\left(x_{(i)} - \sigma\right)$ and $T_{pf}(\underline{x},\sigma) = \sum_{i=1}^{m} (R_i + k)(x_{(i)} - \sigma)^2$.

Remarks:

(1). It is noted that when location parameter is zero (i.e., $\sigma = 0$); $\left(x^2_{(i)}/2\theta^2\right) \forall\ i = 1, 2, \ldots, n,$ is distributed Exponential with mean two and distribution of $\sum_{i=1}^{m} x^2_{(i)} (R_i + 1)$ is Gamma with shape parameter m and scale parameter two.

(2). For $k = 1$, first failure Progressive censoring criterion is converted into conventional Progressive Type - II right censoring criterion.

(3). For $R_i = 0\ \forall\, i = 1, 2, \ldots, m-1$ and $k = 1$, Progressively Type-II right censoring scheme reduces to usual Type - II censoring scheme and for $k = 1, R_i = 0\ \forall\, i = 1, 2, \ldots, m$, the censoring reduces to complete sample case.

3. Bayes estimation when location parameter is known

In present section, the scale parameter θ is considered as a random variable with known location parameter σ. A conjugate family of prior density for θ is considered as an inverted Gamma having probability density function

$$g_1(\theta) \propto \theta^{-2\alpha-1} \exp\left(-\frac{1}{2\theta^2}\right) ; \ \alpha > 0, \theta > 0 . \tag{3.1}$$

There is clearly no way in which one can say that one prior is better than other. It is more frequently the case that, we select to restrict attention to a given flexible family of priors, and we choose one from that family, which seems to match best with our personal beliefs. The prior (3.1) has advantages over many other distributions because of its analytical tractability, richness and easy interpretability.

Following Kundu (2008), the posterior density is defined as

$$\pi(\theta \mid \underline{x}, \sigma) = \frac{f_{X_{1:m:n:k}, X_{2:m:n:k}, \dots, X_{m:m:n:k}}(\theta, \sigma \mid \underline{x}) \cdot g_1(\theta)}{\int_\theta f_{X_{1:m:n:k}, X_{2:m:n:k}, \dots, X_{m:m:n:k}}(\theta, \sigma \mid \underline{x}) \cdot g_1(\theta)\, d\theta} . \tag{3.2}$$

Using (2.2) and (3.1) in (3.2), the posterior density is obtain as

$$\pi(\theta \mid \underline{x}, \sigma) \propto \frac{K_m H_m(\underline{x}, \sigma)\, \theta^{-2m} \exp\left(-\frac{T_{pf}(\underline{x}, \sigma)}{2\theta^2}\right) \cdot \theta^{-2\alpha-1} \exp\left(-\frac{1}{2\theta^2}\right)}{\int_\theta K_m H_m(\underline{x}, \sigma)\, \theta^{-2m} \exp\left(-\frac{T_{pf}(\underline{x}, \sigma)}{2\theta^2}\right) \cdot \theta^{-2\alpha-1} \exp\left(-\frac{1}{2\theta^2}\right) d\theta} .$$

After simplification

$$\pi(\theta \mid \underline{x}, \sigma) = \eta^* \exp\left(-\frac{\hat{T}_{pf}(\underline{x}, \sigma)}{2\theta^2}\right) \theta^{-2(m+\alpha)-1} \tag{3.3}$$

where $\eta^* = \frac{\left(\hat{T}_{pf}(\underline{x}, \sigma)\right)^{m+\alpha}}{\Gamma(m+\alpha)\, 2^{m+\alpha-1}}$ and $\hat{T}_{pf}(\underline{x}, \sigma) = T_{pf}(\underline{x}, \sigma) + 1$.

The selection of loss function may be crucial in Bayesian analysis. It has always been recognized that the most commonly used loss function, squared error loss function (SELF) is inappropriate in many situations. If SELF is taken as a measure of inaccuracy then the resulting risk is often too sensitive to the assumptions about the behavior of tail of the probability distribution. To overcome this difficulty, a useful asymmetric loss function based on SELF is known as invariant squared error loss function (ISELF). Following Prakash (2014), ISELF is defined for any estimate $\hat{\theta}$ corresponding to parameter θ as

$$L(\hat{\theta}, \theta) = \left(\theta^{-1}\partial\right)^2 ; \ \partial = \hat{\theta} - \theta. \tag{3.4}$$

The Bayes estimator corresponding to parameter θ under ISELF is obtained as

$$\hat{\theta}_{I1} = \frac{\int_\theta \theta^{-1} \cdot \pi(\theta \mid \underline{x}, \sigma)\, d\theta}{\int_\theta \theta^{-2} \cdot \pi(\theta \mid \underline{x}, \sigma)\, d\theta}$$

$$\Rightarrow \hat{\theta}_{I1} = \varphi_1 \sqrt{\frac{\hat{T}_{pf}(\underline{x}, \sigma)}{2}} \;;\; \varphi_1 = \frac{\Gamma(m + \alpha + 2^{-1})}{\Gamma(m + \alpha + 1)}. \tag{3.5}$$

When positive and negative errors have different consequences, the use of SELF in Bayesian estimation may not be appropriate. In addition, in some estimation problems overestimation is more serious than the underestimation, or vice-versa. To deal with such cases, a useful and flexible class of asymmetric loss function, LINEX loss function (LLF) is given as

$$L(\partial^*) = e^{a\partial^*} - a\,\partial^* - 1 \;; a \neq 0, \partial^* = (\theta^{-1}\partial).$$

The shape parameter of LLF is denoted by $'a'$. Positive (negative) value of $'a'$, gives more weight to overestimation (underestimation) and its magnitude reflects the degree of asymmetry. It is also seen that, for $a = 1$, the function is quite asymmetric with overestimation being more costly than underestimation. For small values of $|a|$, the LLF is almost symmetric and is not far from SELF.

Bayes estimator $\hat{\theta}_{L1}$ of θ under LLF is obtained by simplifying following equality

$$\int_\theta \theta^{-1} \exp\left(a \frac{\hat{\theta}_{L1}}{\theta}\right) \cdot \pi(\theta \mid \underline{x}, \sigma)\, d\theta = e^{a} \int_\theta \theta^{-1} \cdot \pi(\theta \mid \underline{x}, \sigma)\, d\theta \tag{3.6}$$

$$\Rightarrow \int_\theta \exp\left\{a \frac{\hat{\theta}_{L1}}{\theta} - \frac{\hat{T}_{pf}(\underline{x}, \sigma)}{2\theta^2}\right\} \theta^{-2(m+\alpha+1)}\, d\theta = \frac{e^{a}}{2} \Gamma(m + \alpha + 2^{-1}) \left(\frac{2}{\hat{T}_{pf}(\underline{x}, \sigma)}\right)^{(m+\alpha+2^{-1})}.$$

The close form of Bayes estimator $\hat{\theta}_{L1}$ does not exist. A numerical method is applied here for obtaining the values of the estimates.

4. Bayes estimation when location parameter is unknown

The joint probability density function under Progressive first failure censoring criterion is given in equation (2.2). It is clear from equation (2.2) that, the function $H_m(\underline{x}, \sigma)$ and $T_{pf}(\underline{x}, \sigma)$ both depend upon location parameter σ. Hence, when both parameters are considered as random variables, the joint prior density for parameter θ and σ is defined as

$$g(\theta, \sigma) = g_2(\theta \mid \sigma) \cdot g_3(\sigma). \tag{4.1}$$

Here $g_2(\theta \mid \sigma)$ and $g_3(\sigma)$ are the inverted gamma densities and defined as

$$g_2(\theta \mid \sigma) = \frac{\theta^{-2\sigma-1}\, e^{-1/2\theta^2}}{\Gamma(\sigma)\, 2^{\sigma-1}} \;;\; \theta > 0, \sigma > 0, \tag{4.2}$$

and

$$g_3(\sigma)=\frac{\sigma^{-2\beta-1}\,e^{-1/2\sigma^2}}{\Gamma(\beta)\,2^{\beta-1}}\,;\ \sigma>0,\beta>0. \qquad (4.3)$$

Thus the joint posterior density function is now obtained as

$$\pi^*(\theta,\sigma\,|\,\underline{x})=\overline{\overline{\sigma}}\left(\theta^{-2(\sigma+m)-1}\exp\left(-\frac{\hat{T}_{pf}(\underline{x},\sigma)}{2\theta^2}\right)\right)\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^{\sigma}}\,\sigma^{-2\beta-1}\exp\left(-\frac{1}{2\sigma^2}\right)\right);$$

where $\overline{\overline{\sigma}}=\frac{1}{2^{m-1}\overline{\sigma}}$ and $\overline{\sigma}=\int_{\sigma}\frac{\Gamma(m+\sigma)}{\Gamma(\sigma)}\frac{H_m(\underline{x},\sigma)}{\left(\hat{T}_{pf}(\underline{x},\sigma)\right)^{m+\sigma}}\,\sigma^{-2\beta-1}\exp\left(-\frac{1}{2\sigma^2}\right)d\sigma$.

On similar lines, the Bayes estimator corresponding to the parameter θ under ISELF and LLF are obtained by solving following equality

$$\hat{\theta}_{I2}=\frac{\int_{\sigma}\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)}\,\sigma^{-2\beta-1}\exp\left(-\frac{1}{2\sigma^2}\right)\frac{\Gamma\left(\sigma+m+2^{-1}\right)}{\left(\hat{T}_{pf}(\underline{x},\sigma)\right)^{\sigma+m+1/2}}\,d\sigma}{\sqrt{2}\int_{\sigma}\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)}\,\sigma^{-2\beta-1}\exp\left(-\frac{1}{2\sigma^2}\right)\frac{\Gamma\left(\sigma+m+1\right)}{\left(\hat{T}_{pf}(\underline{x},\sigma)\right)^{\sigma+m+1}}\,d\sigma}$$

and

$$\int_{\sigma}\left(\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)\,2^{\sigma}}\,\sigma^{-2\beta-1}\exp\left(-\frac{1}{2\sigma^2}\right)\right)\left(\int_{\theta}\left(\theta^{-2(\sigma+m)-2}\exp\left(-\frac{\hat{T}_{pf}(\underline{x},\sigma)}{2\theta^2}+a\frac{\hat{\theta}_{L2}}{\theta}\right)\right)d\theta\right)d\sigma$$
$$=\frac{e^a}{2^{m-1/2}}\int_{\sigma}\frac{H_m(\underline{x},\sigma)}{\Gamma(\sigma)}\,\sigma^{-2\beta-1}\exp\left(-\frac{1}{2\sigma^2}\right)\frac{\Gamma\left(\sigma+m+2^{-1}\right)}{\left(\hat{T}_{pf}(\underline{x},\sigma)\right)^{\sigma+m+1/2}}\,d\sigma.$$

The close forms of Bayes estimators $\hat{\theta}_{I2}$ and $\hat{\theta}_{L2}$ do not exist. A numerical method is applied here for obtaining the values of its estimates.

5. Numerical illustration

In the present section, we carry out a comparative analysis based on Bayes estimators under both risk criteria in terms of Relative Efficiency. If $R_{(I)}$ and $R_{(L)}$ are the Bayes risks corresponding to ISELF and LLF risk criteria respectively for the Bayes estimators. Then, the Relative Efficiency under ISELF and LLF for both Bayes estimators are defined as

$$RE_{(I)}=\frac{R_{(I)}\left(\hat{\theta}_{Ii}\right)}{R_{(I)}\left(\hat{\theta}_{Li}\right)} \qquad \text{and} \qquad RE_{(L)}=\frac{R_{(L)}\left(\hat{\theta}_{Ii}\right)}{R_{(L)}\left(\hat{\theta}_{Li}\right)};\ i=1,2.$$

5.1. When location parameter is known

(1). For pre assumed values of the prior parameter α, the random values of θ is generated from prior density (3.1).

(2). We take $k=1$, thus without loss of generality the first failure progressive censoring criterion is converted into Progressive Type - II censoring criterion. Single group size is considered here only for convenience.

(3). Using generated values of θ obtained in steps (1), generates progressively Type - II censored Rayleigh data, for known location parameter σ, of size m for a given censoring scheme $R_i; i=1,2,...,m$, according to an algorithm proposed by Balakrishnan and Aggarwala (2000). The censoring scheme for different values of m is presented in Table 1.

(4). The RE's under both risk criteria are calculated form 1,00,000 generated future ordered samples each of size $N=30$ from Rayleigh model.

(5). The selected values of location parameter and prior parameter are $\sigma = \alpha = 0.50$, 1.00, 5,10 . The value of shape parameter of LLF is taken as $a = 0.25, 0.50, 1.00$. RE's under ISELF and LLF are presented in the Tables 2 - 3 respectively only for $(\alpha = 0.50, 10)$.

Case	m	$R_i; i=1,2,...,m$
1	10	1 2 1 0 0 1 2 0 0 0
2	10	1 0 0 3 0 0 1 0 0 1
3	20	1 0 2 0 0 1 0 2 0 0 0 1 0 0 0 1 0 0 1 0

Table 1: Censoring Scheme for Different Values of m

$N=30$			$\leftarrow \sigma \rightarrow$			
$\alpha \downarrow$	$a \downarrow$	$m \downarrow$	0.50	1.00	5.00	10.00
0.50	0.25	10	1.2282	1.3620	1.6749	1.5104
		10	1.3083	1.4509	1.7844	1.6090
		20	1.3573	1.5052	1.8512	1.6693
	0.50	10	1.4860	1.6479	2.0264	1.8274
		10	1.5829	1.7554	2.1589	1.9467
		20	1.6422	1.8211	2.2397	2.0196
	1.00	10	1.2039	1.3350	1.6419	1.4805
		10	1.2744	1.4133	1.7380	1.5673
		20	1.3247	1.4690	1.8067	1.6291
10	0.25	10	1.1969	1.3273	1.6322	1.4719
		10	1.2749	1.4139	1.7389	1.5680
		20	1.3227	1.4668	1.8040	1.6267
	0.50	10	1.4481	1.6059	1.9747	1.7808
		10	1.5425	1.7107	2.1039	1.8971
		20	1.6003	1.7747	2.1826	1.9681
	1.00	10	1.1732	1.3010	1.6000	1.4428
		10	1.2419	1.3773	1.6937	1.5273
		20	1.2909	1.4316	1.7606	1.5876

Table 2: Relative Efficiency Between $\hat{\theta}_{I1}$ and $\hat{\theta}_{L1}$ under ISELF

N = 30			← σ →			
α ↓	a ↓	m ↓	0.50	1.00	5.00	10.00
0.50	0.25	10	1.4339	1.5901	1.9556	1.7634
		10	1.5179	1.6833	2.0701	1.8668
		20	1.5778	1.7497	2.1519	1.9404
	0.50	10	1.7848	1.9792	2.4338	2.1948
		10	1.9012	2.1084	2.5930	2.3381
		20	1.9724	2.1873	2.6900	2.4257
	1.00	10	1.4087	1.5621	1.9212	1.7323
		10	1.4912	1.6537	2.0336	1.8339
		20	1.5500	1.7189	2.1140	1.9062
10	0.25	10	1.3933	1.5451	1.9003	1.7135
		10	1.4749	1.6357	2.0115	1.8140
		20	1.5331	1.7002	2.0910	1.8855
	0.50	10	1.7066	1.8925	2.3271	2.0986
		10	1.8179	2.0160	2.4793	2.2356
		20	1.8859	2.0914	2.5721	2.3194
	1.00	10	1.3321	1.4771	1.8167	1.6381
		10	1.4101	1.5638	1.9230	1.7342
		20	1.4657	1.6254	1.9990	1.8025

Table 3: Relative Efficiency Between $\hat{\theta}_{I1}$ and $\hat{\theta}_{L1}$ under LLF

It is observed from the tables that the relative efficiency increases when σ increase (for $\sigma \le 5$) and decreases for higher values of σ. Similar behavior has been seen when α increases. However, the difference in magnitude of the relative efficiency is least. It is noted also that when censoring scheme m increases, the relative efficiency also increases. The relative efficiency increases when shape parameter 'a' increases (for $a \le 0.50$) and decreases for $a = 1.00$.

Similar behavior has seen under both risk criteria. However, magnitude of the relative efficiency is higher under LLF as compared to ISELF. On basis of the relative efficiency, one may say that the Bayes estimator $\hat{\theta}_{L1}$ performs uniformly better than Bayes estimator $\hat{\theta}_{I1}$ for all selected parametric values under Progressive first failure censoring criterion for single group case.

5.2. When location parameter is unknown

In case, when both parameters are considered to be random variables, the relative efficiencies are obtained as follows:

(1) Generate location parameter σ from (4.3) for pre-assumed prior values of parameter $\beta\,(= 0.50, 1.00, 5, 10)$. With the help of generated values of σ, generate the values of parameter θ from (4.2).

N = 30			← β →			
α ↓	a ↓	m ↓	0.50	1.00	5.00	10.00
0.50	0.25	10	1.4178	1.5723	1.9335	1.7436
		10	1.5103	1.6749	2.0599	1.8574
		20	1.5668	1.7376	2.1370	1.9270
	0.50	10	1.6813	1.8645	2.2927	2.0676
		10	1.7910	1.9861	2.4427	2.2026
		20	1.8580	2.0605	2.5341	2.2851
	1.00	10	1.3391	1.4849	1.8263	1.6468
		10	1.4175	1.5720	1.9332	1.7433
		20	1.4735	1.6340	2.0096	1.8121
10	0.25	10	1.3565	1.5043	1.8499	1.6682
		10	1.4450	1.6025	1.9709	1.7771
		20	1.4991	1.6625	2.0446	1.8437
	0.50	10	1.5918	1.7653	2.1707	1.9576
		10	1.6957	1.8804	2.3127	2.0854
		20	1.7591	1.9508	2.3992	2.1635
	1.00	10	1.2202	1.3531	1.6642	1.5006
		10	1.2917	1.4325	1.7616	1.5885
		20	1.3427	1.4890	1.8312	1.6512

Table 4: Relative Efficiency Between $\hat{\theta}_{I2}$ and $\hat{\theta}_{L2}$ under ISELF

(2) Following Steps (2) to (5) of subsection (5.1), the relative efficiencies $\hat{\theta}_{I2}$ with respect to $\hat{\theta}_{L2}$ under both risks criteria have been obtained and presented in Tables 4 and 5 respectively only for $(\alpha = 0.50, 10)$.

All the properties are seen to be similar as compared to known case of location parameter. However, the gains in magnitude in RE's are widesr. Again, on basis of relative efficiency, one may say that the Bayes estimator $\hat{\theta}_{L2}$ performs uniformly better than Bayes estimator $\hat{\theta}_{I2}$ for all selected parametric values under Progressive first failure censored data.

N = 30			← β →			
α ↓	a ↓	m ↓	0.50	1.00	5.00	10.00
0.50	0.25	10	1.8471	2.0484	2.5189	2.2715
		10	1.9676	2.1820	2.6836	2.4198
		20	2.0412	2.2637	2.7841	2.5105
	0.50	10	2.1399	2.3731	2.9181	2.6316
		10	2.2796	2.5279	3.1090	2.8034
		20	2.3648	2.6226	3.2254	2.9084
	1.00	10	1.6488	1.8283	2.2487	2.0277
		10	1.7453	1.9356	2.3803	2.1465
		20	1.8143	2.0119	2.4744	2.2312

10	0.25	10	1.8103	2.0076	2.4688	2.2263
		10	1.9285	2.1386	2.6302	2.3717
		20	2.0006	2.2187	2.7287	2.4605
	0.50	10	2.0351	2.2569	2.7752	2.5027
		10	2.1680	2.4041	2.9568	2.6661
		20	2.2490	2.4942	3.0675	2.7660
	1.00	10	1.5221	1.6878	2.0758	1.8718
		10	1.6111	1.7868	2.1973	1.9815
		20	1.6748	1.8572	2.2842	2.0597

Table 5: Relative Efficiency Between $\hat{\theta}_{I2}$ and $\hat{\theta}_{L2}$ under LLF

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