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POSTERIOR ANALYSIS OF A COMPETING RISK MODEL BASED ON DECREASING FAILURE RATE WEIBULL AND EXPONENTIAL FAILURES

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Abstract

The paper considers a competing risk model based on decreasing failure rate Weibull and constant failure rate exponential models. The failure may arise due to either of the two causes where the former represents death due to birth defect and the latter represents an accidental failure that may occur at any moment during the normal life cycle. The Bayes analysis is done using weak but proper priors for the parameters. Since the posterior analysis involves analytically intractable integrals, the paper proposes a Gibbs-Metropolis hybridization scheme to draw the corresponding posterior samples. For initial values of model parameters, the paper proposes the use of maximum likleihood estimates obtained using expectation-maximization algorithm. The numerical illustration is provided based on a simulated data example.

Key Words: Weibull Model, Exponential Model, Decreasing Failure Rate, Constant Failure Rate, Gibbs Sampler, Metropolis Algorithm, Expectation-Maximization Algorithm.

1. Introduction

In life time data analysis both exponential and Weibull distributions are exceedingly entertained models perhaps because of their ease and high usability. Mann et al. (1974), Lawless (2002), Hamada et al. (2008) are some of the important references dealing with variety of developments related to the two models. Whereas Hamada et al. (2008) is exclusively a Bayesian reference, the other two deal with mainly classical inferential developments. In its simplest form, the Weibull distribution can have two parameters with probability density function (pdf) given by,

$$f_Z(z \mid \theta_2, \beta) = \frac{\beta}{\theta_2}\left(\frac{z}{\theta_2}\right)^{\beta-1} exp\left[-\left(\frac{z}{\theta_2}\right)^{\beta}\right]; \quad z > 0, \quad \theta_2, \beta > 0, \tag{1}$$

where Z is used to denote the corresponding random variate, θ_2 is the scale parameter and β determines the shape of the distribution. It is, in fact, the parameter β that makes the Weibull distribution a rich and a flexible family. Reliability characteristics such as the hazard rate, reliability, mean time to failure, etc. are available in closed forms and these are given, respectively, by

$$h(z) = \frac{\beta}{\theta_2}\left(\frac{z}{\theta_2}\right)^{\beta-1},$$

$$R(z) = exp\left[-\left(\frac{z}{\theta_2}\right)^{\beta}\right],$$

$$MTF_Z = \theta_2\Gamma(1+1/\beta).$$

Thus for $\beta > 1$, the hazard rate increases and the model can be used to characterize failure due to ageing. Similarly, for $\beta < 1$, the hazard rate of the model decreases and the situation can be used to characterize failure due to early birth defects or infancy. For $\beta = 1$, the model reduces to one-parameter exponential distribution that characterizes constant hazard rate scenario. This last situation may be attributed to the failures arising due to accidents and it can occur at any time in any given situation either characterized by $\beta > 1$ or $\beta < 1$. It is to be noted that shape of the Weibull distribution is concave and skewed for $\beta > 1$ and convex (exponential type) otherwise. As a result, the inferential developments for the two situations are not exactly the same, see Mann et al. (1974), Upadhyay et al. (2008), etc.

It is often pertinent to consider a situation where failures may occur due to more than one mutually exclusive causes say, for example, $X = min(Y, Z)$ where the random variables X, Y, Z represent failure times. Such a model may be referred to as the competing risk model. Among various possibilities for Y and Z, suppose Z is governed by Weibull Law (1) with $\beta < 1$ and Y is governed by the exponential law representing failures due to accidents. Competing risk models based on exponential and Weibull failures have already been considered in the literature by a number of authors. Friedman and Gertsbakh (1980) is perhaps the earliest reference where the authors have considered mostly the classical inferences. Bosquet et al. (2006) is another important reference based on exponential and Weibull failures with inferential developments mostly in Bayesian framework. The authors have considered a detailed study although restricted to $\beta > 1$ for the Weibull shape parameter, a situation that describes failures due to ageing only. The authors have mentioned in their work that they are deliberately considering $\beta > 1$ but did not provide any logical justification for not entertaining the situation when $\beta < 1$. Among other notable references on competing risk models, one can consider Chan and Meeker (1999), Park and Pedgett (2004), etc. although these references consider competing risk models based on a large family of distributions and not directly concerned with exponential-Weibull based competing risk models. A few other significant references in competing risk analysis involving Weibull or related models include Berger and Sun (1993), Bacha et al. (1998) and Basu et al. (2003), etc. Sinpurwalla (2006) is a recent text covering topic on competing risk analysis as well although the discussion focuses on Bayesian developments only.

This paper is an attempt to the fill in the gap by considering a competing risk model based on the minimum of Weibull (with $\beta < 1$) and exponential random variables. If X is used to denote the corresponding random variable, the hazard rate and the reliability functions for the model can be defined, respectively, as

$$h_X(x) = \frac{1}{\theta_1} + \frac{\beta}{\theta_2}\left(\frac{x}{\theta_2}\right)^{\beta-1}, \qquad (2)$$

$$R_X(x) = \exp\left[-\frac{x}{\theta_1} - \left(\frac{x}{\theta_2}\right)^{\beta}\right], \qquad (3)$$

where scale parameter θ_1 results from the exponential model that can be obtained by taking β=1 in (1). A different scale for the exponential model is used for generality only. Obviously, the pdf corresponding to random variable X can be written as

$$f_X(x|\theta_1,\theta_2,\beta) = \left[\frac{1}{\theta_1} + \frac{\beta}{\theta_2}\left(\frac{x}{\theta_2}\right)^{\beta-1}\right] \exp\left[-\left\{\frac{x}{\theta_1} + \left(\frac{x}{\theta_2}\right)^{\beta}\right\}\right];$$

$$x > 0,\ \theta_1,\theta_2 > 0,\ 0 < \beta < 1, \qquad (4)$$

where θ_1 and θ_2 are the scale parameters and β is the shape parameter. It may be noted that the shape of the distribution is similar to Weibull (with $\beta < 1$) and exponential models though the curve corresponding to (4) passes somewhere between the two curves. The mean time to failure is not available in closed form although it can be solved numerically. The corresponding expression for the mean time to failure is given by

$$MTF_X = \int_0^\infty \exp\left[-\left(\frac{x}{\theta_1}\right) - \left(\frac{x}{\theta_2}\right)^{\beta}\right] dx. \qquad (5)$$

Sometimes, one may also be interested to know the probability of failures arising due to birth defect or infancy over the accidental failures. The corresponding expression though not available in closed form can be worked out by means of numerical integration or Monte Carlo integration. The expression for this probability can be obtained as

$$P(X = Z) = P(Z \leq Y) = 1 - \frac{1}{\theta_1}\int_0^\infty \exp\left[-\left(\frac{y}{\theta_1}\right) - \left(\frac{y}{\theta_2}\right)^{\beta}\right] dy. \qquad (6)$$

The model given in (4) with β restricted to less than unity can be considered as a new model and practically nothing appears in the literature with regard to both classical and Bayesian inferences. At first sight it appears a simple three-parameter family but the inferential developments are slightly difficult compared to its two component models. We do not intend to provide a complete inferential development rather propose a full Bayesian analysis using an important sample based approach. As an intermediate step, we shall also obtain maximum likelihood (ML) estimator using expectation-maximization (EM) algorithm.

The plan of the paper is as follows. The next section considers the Bayesian model formulation for the proposed model (4) using proper but weak priors for the parameters. The section also provides a brief discussion on the corresponding posterior analysis using a hybrid scheme based on the Gibbs and the Metropolis algorithms. This scheme actually uses the Gibbs sampler algorithm but the corresponding full conditionals are generated using Metropolis steps. Also, since the implementation of hybrid scheme requires initial values of the parameters, it is proposed to use ML estimates using EM algorithm. The same has been discussed in brief in subsection 2.1. Section 3 provides numerical illustration based on a simulated data from the model (4). A few numerical estimates are given for illustration presuming that other characteristics of interest can be similarly worked out. A brief conclusion is given in the last section.

2. Bayesian Model Formulation

Let us consider a random sample $\underline{x} : x_1, x_2, ..., x_n$ of size n from the model (4), the corresponding likelihood function (LF) can be written as

$$L(\underline{x}; \theta_1, \theta_2, \beta) = \prod_{i=1}^{n} \left[\frac{1}{\theta_1} + \frac{\beta}{\theta_2} \left(\frac{x_i}{\theta_2} \right)^{\beta-1} \right] \exp\left[-\left\{ \frac{x_i}{\theta_1} + \left(\frac{x_i}{\theta_2} \right)^{\beta} \right\} \right]. \quad (7)$$

The next important task in Bayesian modeling formulation is prior specification for the parameters. Prior distribution does play a crucial role in Bayesian inferences and an inappropriately specified prior may lead to poor inferences. We, however, propose the use of weak but proper priors for the parameters (see also Ranjan et al. (2013)) so that the inferences are mostly data driven. In the same very spirit, we consider independent uniform priors for both scale and shape parameters arising because of the Weibull component. The corresponding choices can be written as

$$\pi(\beta) = U(0,1), \quad (8)$$

$$\pi(\theta_2) = U(0,M), \quad (9)$$

where M is the hyperparameter in the prior for θ_2, a large choice of which increases the vagueness in the prior. Moreover, since β is restricted to less than unity, a choice of uniform prior for β in the range (0, 1) appears natural.

For θ_1, we propose to consider inverted gamma prior with scale parameter a and shape parameter b as given below.

$$\pi(\theta_1) = \frac{a^b \exp(-a/\theta_1)}{(\theta_1)^{b+1}}. \quad (10)$$

There is no specific criterion for the selection of inverted gamma family except that it is flexible and offers a natural conjugate prior for the exponential scale. One can, of course, consider its hyperparameters a and b in such a way that prior remains vague (see, for example, Upadhyay et al. (2001)). We, however, propose to consider a different strategy based on the expert opinion for selection of its hyperparameters (see, for example, Bosquet et al. (2006)). Suppose an expert is asked to give a possible interval for θ_1 where its values are expected to lie and suppose he suggests θ_{10} and θ_{11} as the lower and upper limits of this interval. He further suggests that all the values within this interval are equally probable. Using such a consideration given by the expert, the prior hyperparameters a and b can be obtained using the following relationships.

$$a = \frac{2(\theta_{10}+\theta_{11})(\theta_{10}^2+\theta_{11}^2+\theta_{10}\theta_{11})}{(\theta_{11}-\theta_{10})^2}, \tag{11}$$

$$b = 2 + \frac{3(\theta_{10}+\theta_{11})^2}{(\theta_{11}-\theta_{10})^2}. \tag{12}$$

Of course, larger the difference between θ_{11} and θ_{10}, greater is the vagueness in the prior for θ_1.

Once the priors are specified, the joint posterior of the parameters can be easily obtained by combining the priors with the LF via Bayes theorem. The same can be specified up to proportionality as

$$p(\theta_1,\theta_2,\beta|\underline{x}) \propto \frac{1}{\theta_1^{b+1}} \prod_{i=1}^{n} \left[\frac{1}{\theta_1} + \frac{\beta}{\theta_2}\left(\frac{x_i}{\theta_2}\right)^{\beta-1}\right] \exp\left[-\left\{\frac{a}{\theta_1} + \sum_{i=1}^{n}\left(\frac{x_i}{\theta_1} + \left(\frac{x_i}{\theta_2}\right)^{\beta}\right)\right\}\right]. \tag{13}$$

The posterior given in (13) is analytically difficult to offer closed form solution and, therefore, sample based approaches appear to be the only easy alternative. We proceed by implementing the Gibbs sampler algorithm, the simplest form of which requires generating from one dimensional full conditional at a time by considering all such full conditionals in turn. The complete algorithm and its necessary implementation details can be found in Upadhyay and Smith (1994), among others. To look on the possibility for implementing the algorithm, let us write the three full conditionals as

$$p(\theta_1|\theta_2,\beta,\underline{x}) \propto \frac{1}{\theta_1^{b+1}} \prod_{i=1}^{n} \left[\frac{1}{\theta_1} + \frac{\beta}{\theta_2}\left(\frac{x_i}{\theta_2}\right)^{\beta-1}\right] \exp\left[-\left\{\frac{a}{\theta_1} + \sum_{i=1}^{n}\frac{x_i}{\theta_1}\right\}\right], \tag{14}$$

$$p(\theta_2|\theta_1,\beta,\underline{x}) \propto \prod_{i=1}^{n} \left[\frac{1}{\theta_1} + \frac{\beta}{\theta_2}\left(\frac{x_i}{\theta_2}\right)^{\beta-1}\right] \exp\left\{-\sum_{i=1}^{n}\left(\frac{x_i}{\theta_2}\right)^{\beta}\right\}, \tag{15}$$

$$p(\beta|\theta_1,\theta_2,\underline{x}) \propto \prod_{i=1}^{n} \left[\frac{1}{\theta_1} + \frac{\beta}{\theta_2}\left(\frac{x_i}{\theta_2}\right)^{\beta-1}\right] \exp\left\{-\sum_{i=1}^{n}\left(\frac{x_i}{\theta_2}\right)^{\beta}\right\}. \tag{16}$$

Once the full conditionals are specified, the next objective is to simulate samples from the same so that the Gibbs sampler algorithm can be implemented to get samples from the posterior (13). The details for simulating from the full conditionals are discussed in subsection 2.1.

2.1 A hybrid scheme based on Metropolis within Gibbs sampler

It can be seen that the full conditionals in (14)-(16) are not easy from the viewpoint of sample generation. We, therefore, propose the use of Metropolis algorithm for simulating from each full conditional and thereby refer the scheme as a hybrid scheme based on Metropolis within Gibbs. It is to be noted that the Metropolis algorithm requires instead the generation from a proposal density and accepts the value with some probability, say $\alpha(\lambda, \lambda')$. To clarify, suppose we want to simulate from $p(\lambda|x)$ using a symmetric kernal $q(\lambda, \lambda')$ where λ is the current realization and λ' is the next generated proposal from $q(\lambda, \lambda')$. The algorithm accepts the value λ' with probability

$$\alpha(\lambda, \lambda') = min\left\{\frac{p(\lambda'|x)}{p(\lambda|x)}, 1\right\}. \tag{17}$$

For simulating from the full conditionals, we recommend to work with the parameterization $\lambda_1 = log(\theta_1)$, $\lambda_2 = log(\theta_2)$ and $\lambda_3 = log(\beta)$ and use normal kernel with mean as the current realization and standard deviation as c times the Hessian based approximation at the current realization where c is some scaling constant often taken to be in the range 0.5 and 1.0 (see, for example, Upadhyay and Smith (1994)). For details about the algorithm and its implementation, one can refer to Smith and Roberts (1993), Upadhyay et al. (2001), among others.

For initial values for running the chain, one can use any properly chosen estimates of various parameters although we have used ML estimates obtained using EM algorithm. The details are given in next subsection.

2.2 ML estimation using EM algorithm

Although it can be proved that the likelihood equations have a unique consistent solution, the direct maximization of LF in (7) may often lead to unstable numerical results (see, for example, Bousquet et al. (2006)). To avoid this, one may visualize the proposed competing risk model in an alternative way as an incomplete data model. This visualization is natural since it is not known which of the two component models is actually responsible for a particular observation though it is known that observations are arising from one of the two models. This incomplete data assumption opens the scope for EM algorithm where the apparent advantage is that one can avoid direct maximization of LF in (7) and the maximization step can be implemented separately for exponential and Weibull models.

The EM algorithm is an iterative procedure to find ML estimate in the presence of missing data (see Dempster et al. (1977)). The algorithm iterates between two steps known as expectation (E) and maximization (M) steps. E step finds the conditional expectation of the missing data given the observed data and the current estimated parameters. It then substitutes these expectations for the missing data while

M step maximizes the expected value of log likelihood using expected values of missing observations (see Little and Rubin (2002)). The procedure continues iteratively unless a stability of ML estimates is achieved.

To clarify the idea, let us first consider a binary variable ϵ that completes the data structure. The complete data can then be written as $o_i = (x_i, \epsilon_i),\ i = 1, ..., n$, where the binary variable $\epsilon_i = 1(0)$ indicates that the associated observation is coming from an exponential (Weibull) model. So the resulting competing risk density can be written as

$$f_X(o_i) = [h_Y(x_i)]^{\epsilon_i} [h_Z(x_i)]^{1-\epsilon_i} R_Y(x_i) R_Z(x_i), \tag{18}$$

where $h_Y(h_Z)$ denotes the hazard function corresponding to exponential (Weibull) model. Similarly, $R_Y(R_Z)$ is the reliability function corresponding to exponential (Weibull) model. Obviously, the log likelihood based on complete data $\underline{\mathbf{o}}$=($o_{1,...,}o_n$) can be written as

$$l(\underline{\mathbf{o}}; \theta_1, \theta_2, \beta) = \sum_{i=1}^{n} \epsilon_i \ln [h_Y(x_i)] + (1-\epsilon_i) \ln [h_Z(x_i)] + \ln [R_Y(x_i)] + \ln [R_Z(x_i)]\,. \tag{19}$$

Let us take $\Theta = \{\theta_1, \theta_2, \beta\}$ and let $\tilde{\Theta}$ denotes its current value. The expected value of log likelihood $Q(\Theta|\tilde{\Theta})$ can be given as

$$Q(\Theta|\widetilde{\Theta}) = \sum_{i=1}^{n} \widetilde{p}_Y(x_i) \ln [h_Y(x_i)] + \widetilde{p}_Z(x_i) \ln [h_Z(x_i)] + \ln [R_Y(x_i)] + \ln [R_Z(x_i)]\,, \tag{20}$$

where $\widetilde{p}_Y(x_i) = P(\epsilon_i = 1|\underline{\mathbf{o}}, \tilde{\Theta}) = \frac{h_Y(x_i)}{h_Y(x_i)+h_Z(x_i)}$ and $\widetilde{p}_Z(x_i) = 1 - \widetilde{p}_Y(x_i)$. Here $\widetilde{p}_Y(\widetilde{p}_Z)$ denotes the probability that the observation $i,\ i = 1, ..., n,$ is coming from the exponential (Weibull) distribution.

The equation (20) forms the E-step of EM algorithm. Moreover, it may be noted that (20) has an additive structure that results from the contribution of both exponential and Weibull distributions. This additive decomposition of (20) facilitates the implementation of M-step in the sense that it maximizes separately the terms corresponding to exponential and Weibull distributions. The exponential term can be maximized by direct differentiation with respect to the exponential parameter θ_1 whereas the Weibull term can be maximized using any of the iterative procedures, say for example, Newton-Raphson method (see Mann et al. (1974), Press et al. (2007)), as it cannot provide closed form differentiations when differentiated with respect to the Weibull parameters θ_2 and β. These two steps can be repeated until the iterating algorithm converges to give the desired ML estimates.

3. Numerical illustration

For numerical illustration, we have considered two simulated data sets of size 100 and 200 from the competing risk model (4). These samples were generated using the parameter values $\theta_1 = 20.0$, $\theta_2 = 10.0$ and $\beta = 0.6$. It is to be noted that these choices of parameter values are arbitrary and meant for the sake of illustration although these values provide higher proportion of failures due to initial birth defect, a situation that appears natural to consider. The probability of failure due to infancy (see (6)) for these sets of parameter values can be obtained as 0.66. The complete list of simulated observations is not given due to paucity of space. The arithmetic average and the corresponding standard deviations for these simulated observations were, however, 6.61(6.19) and 7.47(8.11), respectively, where the values in parentheses correspond to those based on sample of size 200.

We next considered implementing the MCMC algorithm as described in Section 2 using Metropolis within Gibbs steps. For the choice of hyperparameters a and b, we consider the specification of expert as θ_{10} = 1.0 and θ_{11} = 30.0 within which the value of θ_1 is expected to lie. Values of a and b were then obtained using (11) and (12). For the hyperparameter M involved in the prior for θ_2, we considered a few arbitrary values for M such as $12.0, 20.0, 30.0$ and it was seen that there was no appreciable change in the results by the variation in the values of M. The results in the present paper are reported for M=20.0. For initial values for starting the chain, we used ML estimates obtained by EM algorithm as discussed in subsection 2.2. These ML estimates were found to be 14.88(19.35), 12.96(11.26) and 0.62(0.61), respectively, for θ_1, θ_2 and β where the values in parentheses correspond to sample size 200.

We rather worked with the parameterization $\lambda_1 = log(\theta_1)$, $\lambda_2 = log(\theta_2)$ and $\lambda_3 = log(\beta)$ and used normal kernel for generating from each full conditional. As initial values, we considered the means of the normal kernels as derived from the corresponding ML estimates of θ_1, θ_2 and β. The exact variances were difficult to obtain and, therefore, we worked with the numerical approximations by evaluating numerically the second derivatives and evaluating the same at the corresponding ML estimates. A number of choices were made for the scaling constant c between 0.5 to 1.0 and it was noted that c = 0.6 provides a good acceptance probability in each case.

We considered a single long run of the Gibbs chain and the convergence monitoring was done using ergodic averages. It was found at about 7000 iterations though the chain was run beyond that to pick up posterior samples of size 1000 by taking outcomes at a gap of 10. The gap was chosen to make serial correlation negligibly small (see also Upadhyay et al. (2012)).

Table 1 provides posterior summaries of θ_1, θ_2 and β in the form of estimated posterior mean, median, mode and highest posterior density (HPD) interval with coverage probability 0.95. Values in parentheses correspond to those based on sample of size 200. In general, the estimates convey that posterior densities are almost symmetrical in each case with not much variability. This last observation is evident from the values of estimated HPD interval with coverage probability 0.95. We also notice from the estimated posterior summaries that the results are close to the true parameter values that were used to generate the simulated data sets and this difference reduces with the increasing sample size. Besides the posterior summaries on the model parameters, the Table 1 also provides the estimated probability of failures due to infancy or birth defects (see (6)) and the value appears to be quite close to its parametric counterpart.

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Table 1: Posterior summaries of θ_1, θ_2, β based on simulated data of size 100 and 200 (values in parenthesis correspond to sample size 200)

Variates	Mean	Median	Mode	0.95 HPD Interval		$\hat{P}(X = Z)$
θ_1	21.82	21.86	21.80	19.19	24.69	
	(21.33)	(21.23)	(21.28)	(18.75)	(24.19)	
θ_2	10.67	10.63	10.65	9.43	12.07	0.637
	(10.38)	(10.33)	(10.36)	(9.15)	(11.69)	(0.642)
β	0.76	0.75	0.75	0.58	0.94	
	(0.61)	(0.60)	(0.61)	(0.51)	(0.71)	

Table 2 provides a few estimated posterior characteristics of reliability, hazard rate and mean time to failure, the first two estimates are obtained at the mission time t=6.0. The values in the parentheses correspond to those based on sample of size 200. Once again we have given the estimated characteristics in the form of posterior mean, median, mode and HPD interval with coverage probability 0.95. It is to be noted that these reliability characteristics are estimated by forming their samples by substitution using the final posterior samples of θ_1, θ_2 and β (see also Upadhyay et al. (2012)). The posterior density estimates of reliability and hazard functions are also shown in Figures 1-2 in the form of box plots. These figures correspond to simulated data set of size 200. These estimated densities are obtained at several mission times t =2(2)10 (see Figures 1-2), which clearly show that the two estimated characteristics, in general, decrease as the time t increases and this observation appears natural as well. The estimates, in general, exhibit almost symmetrical posterior surfaces for all the three characteristics. The variability does not appear to be high which is evident from the values of the 0.95 HPD limits (see Table 2). A word of final remark: we are not going into various details though any desired posterior characteristic can be studied and conclusions can be accordingly drawn once we have samples from the various posteriors.

Table 2: Posterior summaries of different reliability characteristics based on simulated data of size 100 and 200 (values in parenthesis correspond to sample size 200)

Reliability Characteristic	Mean	Median	Mode	0.95 HPD Interval	
$h_X(t = 6.0)$	0.128	0.127	0.127	0.113	0.141
	(0.120)	(0.119)	(0.120)	(0.109)	(0.131)
$R_X(t = 6.0)$	0.397	0.396	0.396	0.359	0.434
	(0.369)	(0.366)	(0.368)	(0.342)	(0.393)
MTF_X	7.237	7.231	7.232	6.604	7.852
	(7.152)	(7.127)	(7.131)	(6.528)	(7.706)

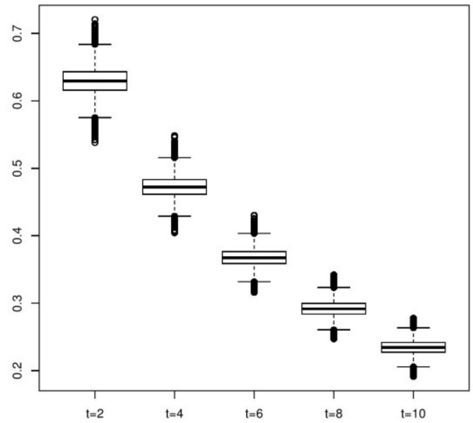

Fig. 1: Boxplot showing the estimated posteriors of reliability for different values of t

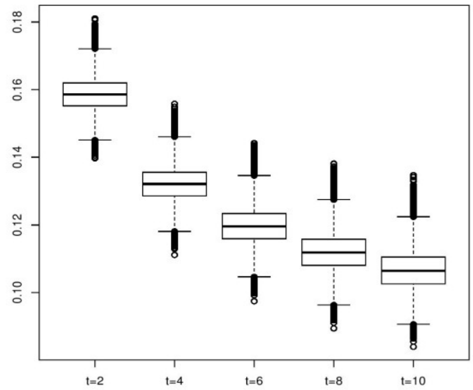

Fig. 2: Boxplot showing the estimated posteriors of hazard function for different values of t

4. Conclusion

The paper successfully considers a simple competing risk model based on Weibull and exponential failures where the former is restricted to shape less than unity. Such models are important because they consider the failures due to initial birth defects and simultaneously take in to account the situation when items are subject to the risk of accidental failures specified by the constant hazard. The paper provides the complete Bayes analysis using Gibbs sampler algorithm with intermediate Metropolis steps for generation from various full conditionals. The results are obtained using simulated data sets of size 100 and 200. ML estimates based on EM algorithm is also attempted which might be of interest to classical statisticians as well though ML estimation is attempted for starting the MCMC chain. Overall, the paper provides yet another interesting scenario of low dimensional posterior and describes the scope of Markov chain Monte Carlo simulation for complete posterior analysis.

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