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AN M/M/1/N FEEDBACK QUEUING SYSTEM WITH REVERSE BALKING

[1]Rakesh Kumar, [2]Bhupender Kumar Som and [3]Sunidhi Jain
[1]School of Mathematics, Shri Mata Vaishno Devi University, Katra, India
[2]Jagan Institute of Management Studies, New Delhi, India
[3]University School of Management, Kurukshetra University, Kurukshetra, India
E Mail: [1]rakesh_stat_kuk@yahoo.co.in; [2]bksoam@live.com

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Abstract

In this paper we develop an M/M/1/N feedback queuing system with reverse balking. Reverse balking is a type of customer behavior according to which an arriving customer joins a system with high probability if he encounters large system size and vice-versa. This behavior of a customer can be observed in many businesses such as investment. Feedback customer in queuing literature refers to a customer who is unsatisfied with incomplete, partial or unsatisfactory service. We derive the steady-state solution of the model and obtain some important measures of performance. Sensitivity analysis of the model is also performed with respect to the parameters involved.

Key Words: Reverse Balking, Feedback, Queuing Theory, Customer Impatience

1. Introduction

In highly competitive and fluid economic environment winning customers trust is of immense importance. When it comes to sensitive businesses like investment, the issue of customer satisfaction becomes more important. Existing number of customers with any firm is an evidence of its strength. In case of businesses like insurance a customer always wishes to join the firm having higher number of policies with it (i.e. higher number of customers or larger system size). The probability that a customer joins a particular firm (by purchasing a policy) varies with existing number of customers with the firm. Larger is the system size (the number of customers with the firm) higher is the probability that an arriving customer joins the firm. This customer behavior can be termed as reverse balking. On the other hand,a customer may not get satisfied with the service and rejoins the firm with some probability. Such a customer can be termed as feedback customer.

The notion of balking appears in queuing theory in the work of [Haight, 1957, 1959]. He studies M/M/1 queue with balking in which there is a greatest queue length at which the arrival will not balk. [Ancker and Gafarian, 1963a, 1963b] study M/M/1/N queuing system with balking and reneging and derive its steady state solution.

Feedback in queuing literature refers to rejoining of queue by an unsatisfied customer due to inappropriate quality of service. Rework in industrial operations is also an example of a queue with feedback. [Takac′s,1963] studies queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time spent in the system by a customer. [Devignon and Disney, 1976] study single server queues with state-dependent feedback. [Shanthakumaranand Thangaraj, 2000] consider a single server feedbackqueue with impatient and feedback customers. They study an M/M/1 queuing model for queue length at arrival epochs and obtain result for stationary distribution, mean and variance of queue length. [Ayyappan et al., 2010] study M/M/1retrial queuing system with loss and feedback under non-pre-emptive priority service by matrix geometric method.

Recently, Kumar et al. (2014a, 2014b) state that when it comes to the business pertaining to investment, more number of customers with a particular firm becomes the attracting factor for investing customers. The probability of joining in such a firm is high. Modeling such a system as a queuing system indicates that the probability of balking will be low when the system size is more and vice-versa, which is balking in reverse sense (called as Reverse Balking).

In this paper, we model investment business with customer impatience and feedback as a queuing system. For example, consider any life insurance company where the purchase of a policy refers to the arrival of a customer in the queuing system (insurance firm), the processed claim refers to as the departure from the queuing system, where the claim processing department is considered as a single server, and finite system capacity (i.e. the number of policies it can accommodate).The claims are processed in order of their arrival (i.e. the queue discipline is FCFS). We incorporate feedback and reverse balking into this model. The model is based on Markovian assumptions.

Rest of the paper is structured as follows: in section 2 the model is described; section 3 deals with mathematical formulation and steady-state solution of the model; in section 4 measures of performance are obtained and the sensitivity analysis of the model is performed; in section 5 the model is concluded.

2. Model description

The model is based on following assumptions:

1. The arrival (purchase of insurance policies) to a queuing system (insurance firm) occur one by one in accordance with a Poisson process with mean rateλ. The inter-arrival times are independently, identically and exponentially distributed with parameterλ.
2. There is a single server (claim processing department) and the policy claims are processed one by one. The service (claim processing) times are independently, identically and exponentially distributed with parameter μ.
3. The capacity of the system (the total number of policies an insurance firm can accommodate) is finite, say N.
4. The policy claims are processed in order of their arrival, i.e. the queue discipline is First-come, First-served.
5. (a)When the system is empty (the start of insurance business), the customers balk (do not purchase policy) with probability q' and may purchase with probability$p' = 1- q'$.

(b)When there is at-least one customer in the system, the customers balk with a probability $\left(1-\frac{n}{N-1}\right)$ and join the system with probability $\left(\frac{n}{N}\right)$.Such kind of balking is referred to as reverse balking.

6. A serviced customer (one who received the claim) may find the service unsatisfactory (not happy with the claim amount or else) and rejoins the system as a feedback customer with probability q_1 or leaves the system (satisfied with the amount of maturity)with $p_1 = (1 - q_1)$. A feedback customer (unsatisfied customer) is allowed to join the queue at the back.

3. Mathematical formulation of the model and its steady-state solution

The model is governed by the following differential-difference equations:

$$\frac{dP_0(t)}{dt} = -\lambda p' P_0(t) + \mu p_1 P_1(t) \ ; \ n = 0 \ (1)$$

$$\frac{dP_1(t)}{dt} = \lambda p' P_0(t) - \left\{\left(\frac{1}{N-1}\right)\lambda + \mu p_1\right\} P_1(t) + (\mu p_1)\, P_2(t); \ n = 1 \ (2)$$

$$\frac{dP_n(t)}{dt} = \left(\frac{n-1}{N-1}\right)\lambda P_{n-1}(t) - \left\{\left(\frac{n}{N-1}\right)\lambda + \mu p_1\right\} P_n(t) + \{\mu p_1\}\, P_{n+1}(t) \ ;$$

$$2 \le n \le N-1 \ (3)$$

$$\frac{dP_N(t)}{dt} = \lambda P_{N-1}(t) - \mu p_1 P_N(t) \ ; n = N \ (4)$$

In steady state$\mathrm{Lim}_{t\to\infty} P_n(t) = P_n$ and $\mathrm{Lim}_{t\to\infty} P_n{}'(t) = 0$

Therefore, the equations (1) to (4) become:

$$0 = -\lambda p' P_0 + \mu p_1 P_1; \ \text{n=0} \ (5)$$

$$0 = \lambda p' P_0 - \left\{\left(\frac{1}{N-1}\right)\lambda + \mu p_1\right\} P_1 + (\mu p_1)\, P_2; \ n = 1 \ (6)$$

$$0 = \left(\frac{n-1}{N-1}\right)\lambda P_{n-1} - \left\{\left(\frac{n}{N-1}\right)\lambda + \mu p_1\right\} P_n + \{\mu p_1\}\, P_{n+1}; \ 2 \le n \le N-1 \ (7)$$

$$0 = \lambda P_{N-1} - \mu p_1 P_N \ ; \ n = N \ \ (8)$$

The equations (5) to (8) are solved iteratively to obtain

$$P_n = \begin{cases} \left\{\dfrac{(n-1)!}{(N-1)^{n-1}} \displaystyle\prod_{r=1}^{n} \dfrac{\lambda}{\mu p_1}\right\} p' P_0; & 1 \le n \le N-1 \\ \left\{\dfrac{(N-2)!}{(N-1)^{N-2}} \displaystyle\prod_{r=1}^{N} \dfrac{\lambda}{\mu p_1}\right\} p' P_0; & n = N \end{cases}$$

Since$\sum_{n=0}^{N} P_n = 1$, therefore we get

$$P_0 + \sum_{n=1}^{N-1} P_n + P_N = 1$$

$$P_0 = \left[1 + \sum_{n=1}^{N-1} \left\{\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\lambda}{\mu p_1}\right\} p' + \left\{\frac{(N-2)!}{(N-1)^{N-2}} \prod_{r=1}^{N} \frac{\lambda}{\mu p_1}\right\} p'\right]^{-1}$$

4. Measures of performance

1. **Expected system size, L_s** (Average number of insurance policies with a firm). Expected number of polices firm may hold can be given by expected system size.

$$L_s = \sum_{n=0}^{N} nP_n = \sum_{n=1}^{N-1} nP_n + NP_N$$

$$L_s = \sum_{n=1}^{N-1} n \left\{\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\lambda}{\mu p_2}\right\} p' P_0 + N \left\{\frac{(N-2)!}{(N-1)^{N-2}} \prod_{r=1}^{N} \frac{\lambda}{\mu p_2}\right\} p' P_0$$

2. **Expected queue length, Lq** (Average number of insurance policies with a firm excluding the one being processed for claim)

$$L_q = \sum_{n=1}^{N} (n-1)P_n = \sum_{n=1}^{N-1} nP_n + NP_N + P_0 - 1$$

$$L_q = \sum_{n=1}^{N-1} n \left\{\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\lambda}{\mu p_2}\right\} p' P_0 + N \left\{\frac{(N-2)!}{(N-1)^{N-2}} \prod_{r=1}^{N} \frac{\lambda}{\mu p_2}\right\} p' P_0 + P_0 - 1$$

3. **Average rate of reverse balking, R_b'** (Mean rate at customers do not purchase policies).

Average rate of reverse balking represents the average number of customers arriving to the firm but not purchasing polices.

$$R_b' = q'\lambda P_0 + \sum_{n=1}^{N-1} \left(1 - \frac{n}{N-1}\right) \lambda P_n$$

$$R_b' = q'\lambda P_0 + \sum_{n=1}^{N-1} \left(1 - \frac{n}{N-1}\right) \lambda \left\{\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\lambda}{\mu p_2}\right\} p' P_0$$

4. Sensitivity analysis

Here, we present the sensitivity analysis of the model. We study the variation in average system size and in average rate of reverse balking with respect to the change in system parameters.

λ	L_s	R_b'
2	0.4344	0.7993
2.5	0.8992	1.5228
3	1.8846	1.4559
3.5	3.3821	1.1621
4	4.9056	0.7828
4.5	6.0243	0.4765
5	6.7105	0.2816
5.5	7.1077	0.1688
6	7.3406	0.1047
6.5	7.4832	0.0677
7	7.5755	0.0457
7.5	7.6387	0.0321
8	7.6841	0.0234
8.5	7.7183	0.0177
9	7.7450	0.0138
9.5	7.7665	0.0110
10	7.7842	0.0090

Table 1: Variation in L_s, R_b' with respect to λ (μ =2, q' =0.8, q_1=0.2, N =8)

From Table 1 it can be observed that expected system size increases with increase in average rate of arrival. The rate to reverse balking increases initially but as soon as the expected system size gets bigger and bigger, people show a high confidence in the system due to that rate of reverse balking reduces. Much clear insight can be observed from following figure (Figure 1). The blue colored curve (i.e. the upper curve) is of L_s and the red colored curve (i.e. the lower curve) represents R_b'.

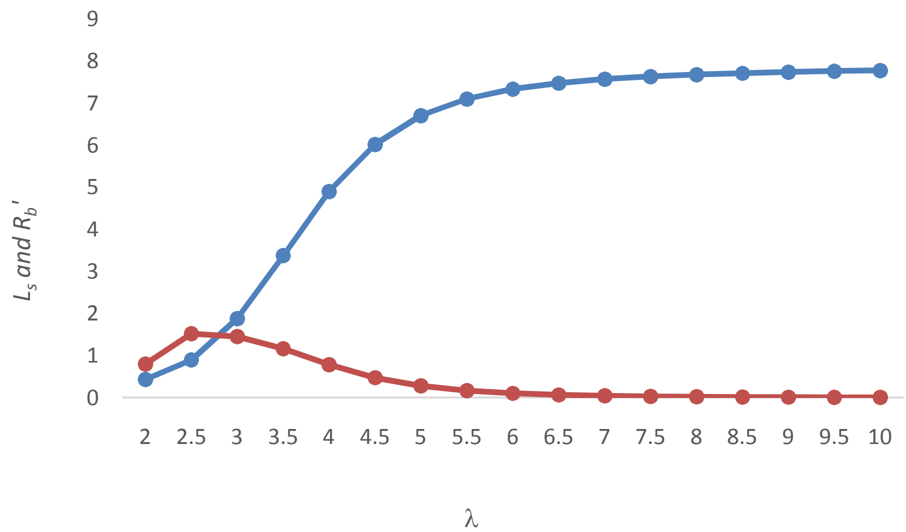

Figure 1: L_s and R_b' w.r.t. λ

μ	L_s	R_b'
1.2	7.7079	0.0162
1.4	7.5959	0.0344
1.6	7.4201	0.0731
1.8	7.1400	0.1486
2	6.7105	0.2816
2.2	6.1027	0.4895
2.4	5.3319	0.7743
2.6	4.4682	1.1144
2.8	3.6123	1.4718
3	2.8497	1.8087
3.2	2.2235	2.1020
3.4	1.7370	2.3442
3.6	1.3710	2.5389
3.8	1.0998	2.6938
4	0.8992	2.8174

Table 2: Variation in L_s, R_b', with respect to μ (λ =5, q' =0.8, q_1 =0.2, N =8)

From Table 2 it can be observed that expected system size decreases with increase in average rate of service μ. Since expected system size decreases, the rate of reverse balking keeps on increasing which shows that reducing number of customers in the system leaves a negative impression on the people. Customers start losing confidence in the particular firm and show less willingness to join it. Following figure (Figure 2) depicts results more clearly. The blue colored curve (i.e. the upper curve) is of L_s and the red colored curve (i.e. the lower curve) represents R_b'.

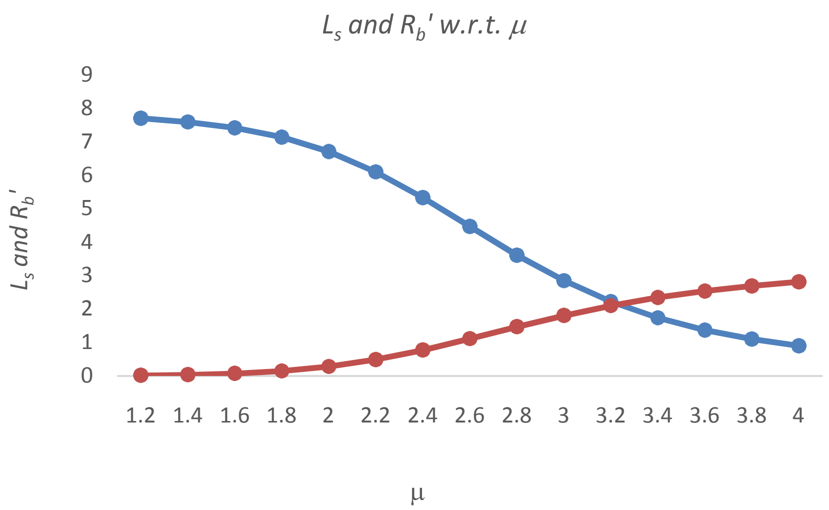

Figure 2

q'	L_s	$R_{b'}$
0.1	6.9952	0.0814
0.2	6.9846	0.0888
0.3	6.9711	0.0984
0.4	6.9531	0.1110
0.5	6.9280	0.1286
0.6	6.8908	0.1548
0.7	6.8296	0.1978
0.8	6.7105	0.2816
0.9	6.3767	0.5163

Table 3: Variation in L_s, R_b' with respect to q' (λ =5, μ =2, q_1=0.2, N =8)

From Table 3 it can be observed that expected system size decreases with increase in rate of reverse balking. This is due to the fact that higher rate of reverse balking means less number of customers are joining the firm. Eventually the rate of reverse balking also increases.

q_1	L_s	R_b'
0.1	5.9234	0.5539
0.2	6.7105	0.2816
0.3	7.2223	0.1252
0.4	7.5183	0.0503
0.5	7.6841	0.0195
0.6	7.7842	0.0079
0.7	7.8541	0.0033
0.8	7.9099	0.0013
0.9	7.9577	0.0003

Table 4: Variation in L_s, $R_b'$with respect to q_1 (λ =5, q' =0.8, μ =2, N =8)

From table-4 it can be observed that expected system size increases when more and more customers rejoin the system as feedback customers. More and more customers in the system also ensure a decreasing rate of reverse balking.

5. Conclusion

In this paper, the concept of reverse balking is incorporated into an M/M/1/N Feedback queuing system. The steady-state analysis of the model is performed and some important measures of performance are derived. Sensitivity analysis of the model is also performed numerically. Output of the model is represented through graphs for better insight. Numerical analysis carried out in this paper validates the function of the model under investigation. This model finds its application in investment business facing customer impatience. The model may be extended to infinite case. Further, time-dependent analysis can also be carried out.

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