Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 8, Issue 1 (2015): 01-13

A TWO UNIT PARALLEL LOAD SHARING SYSTEM WITH ADMINISTRATIVE DELAY IN REPAIR

Alka Chaudhary, Suman Jaiswal and †Rakesh Gupta
Department of Statistics, Meerut College, Meerut, India
†Department of Statistics, C.C.S. University, Meerut, India
E.Mail: †prgheadstats@yahoo.in
†Corresponding Author

Received January 13, 2015
Modified May 21, 2015
Accepted May 27, 2015

Abstract

The paper deals with the stochastic analysis of a two non-identical unit parallel system model assuming that an administrative delay occurs in getting the repairman available with the system whenever needed. Upon failure of a unit, the other operating unit shares the load of previous one. The failure time distributions of the units are assumed to be exponential whereas the repair time distributions of the failed units and administrative delay time distribution are taken general having different c.d.f.'s. By using regenerative point technique, various performance measures of system effectiveness have been obtained.

Key Words: Transition Probabilities, Mean Sojourn Time, MTSF, Availability, Expected Busy Period of Repairman, Net Expected Profit.

1. Introduction

The incorporation of redundancy is one of the important devices to enhance the reliability of a system. Two-unit active (parallel) and standby redundant systems have been widely studied in the literature of reliability due to their frequent and significant use in modern business and industries. A large number of authors including [1-3, 6-10] have analysed the system models pertaining to two identical and non-identical unit active/standby redundant systems under various sets of assumptions by using different techniques. The common assumption considered in all the system models is that a single repair facility is either always available with the system to take up a failed unit immediately for its repair provided the repair facility is not busy in the repair of other unit or he/she becomes available instantaneously whenever required. In earlier situation one is to pay the repairman un-necessarily even though he is not busy in the repair of a failed unit whereas in later case the assumption seems to be un-realistic. In real existing situations, it is very difficult for the repairman to be available instantaneously due to so many reasons. The one particular reason may be due to administrative decisions. In this situation there is an administrative delay in getting the repairman available with the system. Jaiswal and Krishna [5] first introduced the concept of administrative delay in two-unit standby system and obtained only MTSF and steady-state availability of the system. Later on Gupta and Goel [4] analyzed a two-unit priority standby system with administrative delay in repair.

In the present paper we analyse a two non-identical unit parallel system model assuming that an administrative delay occur in getting the repairman available with the system. It is also assumed that the single operating unit shares the work-load of earlier failed unit. In view of this, the single operating unit has increased failure rate as compared to the situation when it works in parallel with other unit. By using regenerative point technique, the following economic related measures of system effectiveness are obtained:

(i) State transition probability and mean sojourn times,
(ii) The reliability of the system and mean time to system failure (MTSF),
(iii) Pointwise and steady-state availabilities of the system due to the operation of both the units, only unit-1 and only unit-2,
(iv) Busy period of repairman in the repair of unit-1 and unit-2 during (0, t),
(v) Expected number of visits by the repairman during time interval (0, t),
(v) Net expected profit earned by the system during time interval (0, t) and in steady state.

The graphical behaviors of MTSF and Profit function have also been studied in Fig. 2 to Fig. 4 for two particular cases in respect of different parameters.

2. System description and assumptions

The system under study is developed under the following assumptions:

(i) The system comprises of two non-identical units which are named as unit-1 and-2. Initially both the units work in parallel configuration.
(ii) Each unit of the system has two modes—Normal (N) and Total failure (F).
(iii) Repairman is not always available with the system. Whenever a unit fails, he is called to visit the system who takes some significant time to come at the system due to some administrative actions.
(iv) The repairman upon arrival repairs the failed unit with general repair time distribution which is different for each unit.
(v) The repair discipline is FCFS as only single repairman can be made available to repair the failed units.
(vi) The failures of the units are independent and the failure time distributions of the units are taken exponentials.
(vii) If one unit has failed then the other unit in N-mode works with increased failure rate as compared to that when both the units work in parallel form.
(viii) The administrative delay time distribution is also assumed to be general.
(ix) Each repaired unit works as good as new.

3. Notations and states of the system

3.1 Notations

E : Set of regenerative states

$\equiv \{S_0, S_1, S_2, S_4, S_5, S_6, S_7, S_8\}$

α_1, α_2	:	Constant failure rates of unit-1 and unit-2 when both units operate in parallel.
$\alpha_1^{'}(>\alpha_1)$	:	Constant failure rate of unit-1 when unit-2 has failed.
$\alpha_2^{'}(>\alpha_2)$	:	Constant failure rate of unit-2 when unit-1 has failed.
$G_1(\cdot), G_2(\cdot)$	:	c.d.f. of repair times of unit-1 and unit-2 respectively. The corresponding small alphabets denote the p.d.fs of repair times.
$h(\cdot), H(\cdot)$	:	p.d.f. and c.d.f. of administrative delay time for getting the repairman available.
$q_{ij}(\cdot), q_{ij}^{(k)}(\cdot)$	:	Direct and indirect (via state S_k) transition time p.d.fs from state S_i to S_j.
ψ_i	:	Mean sojourn time in state S_i.
θ_1, θ_2, ϕ	:	Mean repair times of unit-1, unit-2 and administrative delay time for getting the repairman available.
*, ~	:	Symbols for Laplace and Laplace-Stieltijes transforms.

3.2 Symbols for the states of the system

N_0^1, N_0^2	:	Unit-1 and unit-2 in normal mode and operative.
F_d^1, F_d^2	:	Unit-1 and unit-2 in failure mode and waiting for repairman due to administrative delay.
F_r^1, F_r^2	:	Unit-1 and unit-2 in failure mode and under repair.
F_w^1, F_w^2	:	Unit-1 and unit-2 in failure mode and waiting for repair.

Using the above symbols in view of the assumptions stated in section-2, the possible states of the system are shown in transitions diagram (Fig.1). The epochs of transitions into the states S_4 from S_1, S_6 from S_2, S_7 from S_3 and S_8 from S_5 are non-regenerative whereas all the other entrance epochs into the states are regenerative.

4. Transition probabilities and mean sojourn times

Let $X(t)$ be the state of the system at epoch t, then $\{X(t);\ t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space E. The transition probability matrix of the embedded Markov-Chain is

$$p = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{05} & p_{07} & p_{08} \\ p_{10} & p_{11} & p_{12} & p_{13} & p_{15} & p_{17}^{(4)} & p_{18} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{25} & p_{27} & p_{28}^{(6)} \\ p_{30} & p_{31} & p_{32} & p_{33} & p_{35}^{(7)} & p_{37} & p_{38} \\ p_{50} & p_{51} & p_{52} & p_{53}^{(8)} & p_{55} & p_{57} & p_{58} \\ p_{70} & p_{71} & p_{72} & p_{73} & p_{75} & p_{77} & p_{78} \\ p_{80} & p_{81} & p_{82} & p_{83} & p_{85} & p_{87} & p_{88} \end{bmatrix}$$

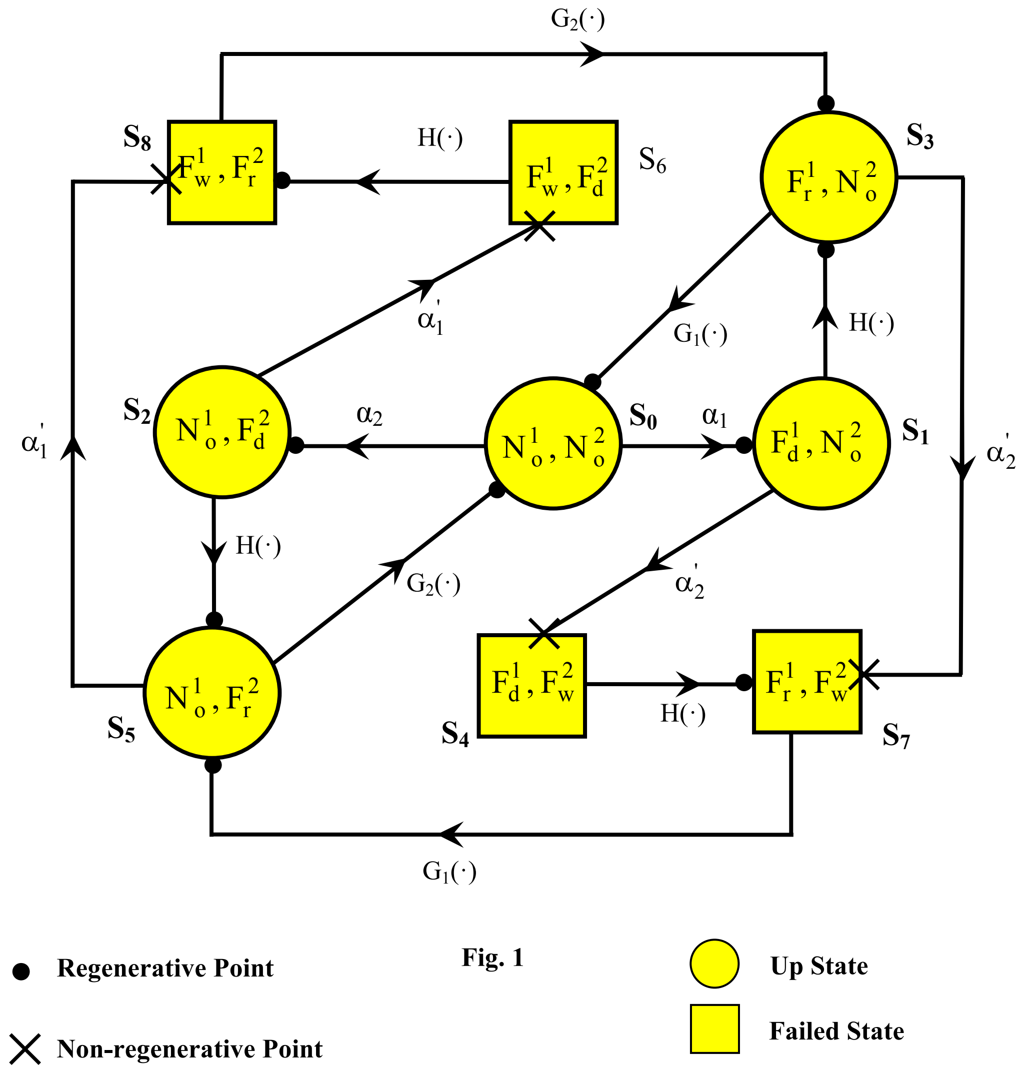

Fig. 1

with non-zero elements:

$$p_{01} = \alpha_1/(\alpha_1+\alpha_2), \quad p_{02} = \alpha_2/(\alpha_1+\alpha_2)$$
$$p_{13} = \tilde{H}(\alpha'_2) \quad , \quad p_{17}^{(4)} = 1-\tilde{H}(\alpha'_2)$$
$$p_{25} = \tilde{H}(\alpha'_1) \quad , \quad p_{28}^{(6)} = 1-\tilde{H}(\alpha'_1)$$
$$p_{30} = \tilde{G}_1(\alpha'_2) \quad , \quad p_{35}^{(7)} = 1-\tilde{G}_1(\alpha'_2)$$
$$p_{50} = \tilde{G}_2(\alpha'_1) \quad , \quad p_{53}^{(8)} = 1-\tilde{G}_2(\alpha'_1) \qquad (1\text{-}10)$$

The other elements of t.p.m. will be zero.

It can be easily verified that

$$p_{01} + p_{02} = 1,\ \ p_{13} + p_{17}^{(4)} = 1$$

$$p_{25} + p_{28}^{(6)} = 1,\ \ p_{30} + p_{35}^{(7)} = 1$$

$$p_{50} + p_{53}^{(8)} = 1 \qquad (11\text{-}15)$$

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then

$$\psi_i = \int P(U_i > t)\,dt$$

Its values for various regenerative state are as follows—

$$\psi_0 = 1/(\alpha_1 + \alpha_2),\ \ \psi_1 = \{1 - \tilde{H}(\alpha'_2)\}/\alpha'_2$$

$$\psi_2 = \{1 - \tilde{H}(\alpha'_1)\}/\alpha'_1,\ \ \psi_3 = \{1 - \tilde{G}_1(\alpha'_2)\}/\alpha'_2$$

$$\psi_5 = \{1 - \tilde{G}_2(\alpha'_1)\}/\alpha'_1,\ \ \psi_7 = \int \tilde{G}_1(t)\}dt = \theta_1$$

$$\psi_8 = \int \tilde{G}_2(t)\}dt = \theta_2 \qquad (16\text{-}22)$$

5. Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t) given that at t = 0 system starts from $S_i \in E$. To obtain it we assume the failed states S_4, S_6, S_7 and S_8 of the system as absorbing states. By using elementary probabilistic arguments in renewal theoretic approach, the following recursive relations among $R_i(t)$; i = 0, 1, 2, 3, 5 can be obtained—

$$R_0(t) = Z_0(t) + q_{01}(t) \copyright R_1(t) + q_{02}(t) \copyright R_2(t)$$

$$R_1(t) = Z_1(t) + q_{13}(t) \copyright R_3(t)$$

$$R_2(t) = Z_2(t) + q_{25}(t) \copyright R_5(t)$$

$$R_3(t) = Z_3(t) + q_{30}(t) \copyright R_0(t)$$

$$R_5(t) = Z_5(t) + q_{50}(t) \copyright R_0(t) \qquad (23\text{–}27)$$

where,

$$Z_0(t) = e^{-(\alpha_1+\alpha_2)t},\ Z_1(t) = e^{-\alpha'_2 t}\,\bar{H}(t)$$

$$Z_2(t) = e^{-\alpha'_1 t}\,\bar{H}(t),\ Z_3(t) = e^{-\alpha'_2 t}\,\bar{G}_1(t)$$

$Z_5(t) = e^{-\alpha'_1 t}\,\bar{G}_2(t)$ Taking L.T. of relations (23-27) and simplifying the resulting set of algebraic equations for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad (28)$$

where,

$$N_1(s) = Z_0^* + q_{01}^* Z_1^* + q_{01}^* q_{13}^* Z_3^* + q_{02}^* q_{25}^* Z_5^*$$

and $D_1(s) = 1 - q^*_{01}\, q^*_{13} q^*_{30} - q^*_{02}\, q^*_{25} q^*_{50}$

Taking the inverse Laplace transform of (28), one can get the reliability of the system when it starts from state S_0. The MTSF is given by

$$E(T_0) = \int R_0(t)\,dt = \lim_{s\to 0} R^*_0(s)$$

$$= \frac{\psi_0 + p_{01}(\psi_1 + p_{13}\psi_3) + p_{02}(\psi_2 + p_{25}\psi_5)}{1 - p_{01}p_{13}p_{30} - p_{02}p_{25}p_{50}} \quad (29)$$

6. Cost-benefit analysis

In order to find the net expected profit earned by the system during interval (0,t) and in steady-state, we compute the following:

6.1 Availability analysis

Let $A^b_0(t)$, $A^1_0(t)$ and $A^2_0(t)$ be the respective probabilities that both the units, only unit-1 and only unit-2 are operative at epoch t when initially system starts functioning from state S_i (i = 0, 1, 2, 3, 5,7, 8). By using elementary probabilistic reasoning with regenerative point technique and the tools of L.T. as in case of reliability analysis, one can obtain the values of $A^b_0(t)$, $A^1_0(t)$ and $A^2_0(t)$ in terms of the their Laplace transforms i.e. $A^{b*}_0(s)$, $A^{1*}_0(s)$ and $A^{2*}_0(s)$.

The steady-state availabilities of the system due to the operation of both the units, only unit-1 and only unit-2 are given by

$$A^b_0 = \frac{N_2}{D_2}; \quad A^1_0 = \frac{N_3}{D_2} \quad \text{and} \quad A^2_0 = \frac{N_4}{D_2} \quad (30\text{-}32)$$

where

$$N_2 = (1 - p^{(7)}_{35} p^{(8)}_{53})\psi_0$$

$$N_3 = (1 - p^{(7)}_{35} p^{(8)}_{53})p_{02}\psi_2 + \{p_{01}(1 - p_{13}p_{30}) + p_{02}(1 - p^{(6)}_{28} p_{30})\}\psi_5$$

$$N_4 = (1 - p^{(7)}_{35} p^{(8)}_{53})p_{01}\psi_1 + \{p_{01}(1 - p^{(4)}_{17} p_{50}) + p_{02}(1 - p_{25}p_{50})\}\psi_3$$

and

$$D_2 = (1 - p^{(7)}_{35} p^{(8)}_{53})(\psi_0 + \phi) + (1 - p_{02}p_{25}p_{50} + p_{01}p^{(4)}_{17} p^{(8)}_{53} p_{30})\theta_1 + (1 - p_{01}p_{13}p_{30} + p_{02}p^{(6)}_{28} p^{(7)}_{35} p_{50})\theta_2 \quad (33)$$

The expected up time of the system due to the operation of both the units, only unit-1 and only unit-2 in time interval (0, t) are as follows—

$$\mu^b_{up}(t) = \int_0^t A^b_0(u)\,du, \quad \mu^1_{up}(t) = \int_0^t A^1_0(u)\,du, \quad \mu^2_{up}(t) = \int_0^t A^2_0(u)\,du \quad (34\text{-}36)$$

6.2 Busy period analysis of repairman

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the server (repairman) is busy at epoch t in the repair of unit-1 and unit-2 when initially system starts operation from state $S_i \in E$. Here again by using the basic probabilistic arguments with regenerative point technique and the tools of L.T. as in case of reliability analysis, one can obtain the values of $B_0^1(t)$ and $B_0^2(t)$ in terms of their L.T. i.e. $B_0^{1*}(s)$ and $B_0^{2*}(s)$.

The steady-state probabilities that the server (repairman) is busy in the repair of unit-1 and unit-2 are given by

$$B_0^1 = \frac{N_5}{D_2} \quad \text{and} \quad B_0^2 = \frac{N_6}{D_2} \tag{37-38}$$

where

$$N_5 = [p_{01}p_{17}^{(4)}(1-p_{35}^{(7)}p_{53}^{(8)})+p_{01}(1-p_{17}^{(4)}p_{50})+p_{02}(1-p_{25}p_{50})]\theta_1$$

$$N_6 = [p_{02}p_{28}^{(6)}(1-p_{35}^{(7)}p_{53}^{(8)})+p_{02}(1-p_{28}^{(6)}p_{30})+p_{01}(1-p_{13}p_{30})]\theta_2$$

The expected busy period of repairman in the repair of unit-1 and unit-2 during (0, t) are

$$\mu_b^1(t) = \int_0^t B_0^1(u)\,du \quad \text{and} \quad \mu_b^2(t) = \int_0^t B_0^2(u)\,du \tag{39-40}$$

6.3 Expected number of visits by repairman

Let $V_i(t)$ be the expected number of visits by the repairman during interval (0, t) when system initially starts from $S_i \in E$. The value of $V_0(t)$ in terms of its L.S.T. $\tilde{V}_0(s)$ can easily be obtained by simple probabilistic arguments in regenerative point technique.

The expected number of visits by the repairman per-unit time in steady-state is given by

$$V_0 = \lim_{t\to\infty} \frac{V_0(t)}{t} = \lim_{s\to 0} s\tilde{V}_0(s) = (1-p_{35}^{(7)}p_{53}^{(8)})/D_2 \tag{41}$$

Now we are in the position to obtain net expected profit earned by the system during interval (0, t) by considering the above characteristics and it will be as follows—

P(t) = Expected total revenue in (0, t) – Expected total cost on repairman during (0, t)

$$= K_0\mu_{up}^b(t)+K_1\mu_{up}^1(t)+K_2\mu_{up}^2(t)-K_3\mu_b^1(t)-K_4\mu_b^4(t)-K_5V_0(t) \tag{42}$$

where, K_0, K_1 and K_2 are the per-unit up time revenues by the system corresponding to the operation of both the units, only unit-1and only unit-2. K_3 and K_4 be the amount spent per-unit of time in repairing the failed unit-1and unit-2 respectively. K_5 be the per-visit amount paid to the repairman.

The expected profit per-unit time in steady-state is given by

$$P = \lim_{t\to\infty} \frac{P(t)}{t} = K_0 A_0^b + K_1 A_0^1 + K_2 A_0^2 - K_3 B_0^1 - K_4 B_0^2 - K_5 V_0 \quad (43)$$

The values of $A_0^b, A_0^1, A_0^2, B_0^1, B_0^2$ and V_0 are shown by expressions (30-32), (37), (38) and (41) respectively.

7. Case studies

The system model has wide applicability for various form of p.d.f./c.d.f. of repair times of unit-1 and unit-2 as well as administrative delay time of repairman. As an illustration, we consider the following two cases to obtain the specific values of various measures of system effectiveness obtained in earlier sections.

Case-I when repair times and administrative delay time follow Lindley distributions with p.d.f as follows:

$$g_1(t) = \frac{\beta_1^2}{(1+\beta_1)}(1+t)e^{-\beta_1 t}\ ;\ t \ge 0 \quad , \qquad g_2(t) = \frac{\beta_2^2}{(1+\beta_2)}(1+t)e^{-\beta_2 t}\ ;\ t \ge 0$$

and $$h(t) = \frac{\lambda^2}{(1+\lambda)}(1+t)e^{-\lambda t}\ ;\ t \ge 0$$

The Laplace Transforms of above three density functions are equivalent to the Laplace-Stieltjes transforms of the corresponding c.d.f. and are given below:

$$g_1^*(s) = \tilde{G}_1(s) = \left(1+\frac{s}{\beta_1+1}\right)\left(\frac{\beta_1}{s+\beta_1}\right)^2,$$

$$g_2^*(s) = \tilde{G}_2(s) = \left(1+\frac{s}{\beta_2+1}\right)\left(\frac{\beta_2}{s+\beta_2}\right)^2$$

and $$h^*(s) = \tilde{H}(s) = \left(1+\frac{s}{\lambda+1}\right)\left(\frac{\lambda}{s+\lambda}\right)^2$$

In view of above, we have the following changes in results (3-10) and (18-22).

$$p_{13} = \left(1+\frac{\alpha'_2}{1+\lambda}\right)\left(\frac{\lambda}{\alpha'_2+\lambda}\right)^2, \qquad p_{17}^{(4)} = 1-\left(1+\frac{\alpha'_2}{1+\lambda}\right)\left(\frac{\lambda}{\alpha'_2+\lambda}\right)^2$$

$$p_{25} = \left(1+\frac{\alpha'_1}{1+\lambda}\right)\left(\frac{\lambda}{\alpha'_1+\lambda}\right)^2, \qquad p_{28}^{(6)} = 1-\left(1+\frac{\alpha'_1}{1+\lambda}\right)\left(\frac{\lambda}{\alpha'_1+\lambda}\right)^2$$

$$p_{30} = \left(1+\frac{\alpha'_2}{1+\beta_1}\right)\left(\frac{\beta_1}{\alpha'_2+\beta_1}\right)^2, \qquad p_{35}^{(7)} = 1-\left(1+\frac{\alpha'_2}{1+\beta_1}\right)\left(\frac{\beta_1}{\alpha'_2+\beta_1}\right)^2$$

$$p_{50} = \left(1+\frac{\alpha'_1}{1+\beta_2}\right)\left(\frac{\beta_2}{\alpha'_1+\beta_2}\right)^2, \qquad p_{53}^{(8)} = 1-\left(1+\frac{\alpha'_1}{1+\beta_2}\right)\left(\frac{\beta_2}{\alpha'_1+\beta_2}\right)^2$$

$$\psi_1 = \frac{1}{\alpha'_2}\left[1-\left(1+\frac{\alpha'_2}{1+\lambda}\right)\left(\frac{\lambda}{\alpha'_2+\lambda}\right)^2\right]$$

$$\psi_2 = \frac{1}{\alpha'_1}\left[1-\left(1+\frac{\alpha'_1}{1+\lambda}\right)\left(\frac{\lambda}{\alpha'_1+\lambda}\right)^2\right]$$

$$\psi_3 = \frac{1}{\alpha'_2}\left[1-\left(1+\frac{\alpha'_2}{1+\beta_1}\right)\left(\frac{\beta_1}{\alpha'_2+\beta_1}\right)^2\right]$$

$$\psi_5 = \frac{1}{\alpha'_1}\left[1-\left(1+\frac{\alpha'_1}{1+\beta_2}\right)\left(\frac{\beta_2}{\alpha'_1+\beta_2}\right)^2\right]$$

$$\theta_1 = \frac{\beta_1+2}{\beta_1(1+\beta_1)}, \qquad \theta_2 = \frac{\beta_2+2}{\beta_2(1+\beta_2)}$$

and the value of mean administrative delay time will be

$$\phi = \frac{\lambda+2}{\lambda(1+\lambda)}$$

Case-II When repair times and administrative delay time follow Gamma distributions with p.d.fs

$$g_1(t) = \frac{e^{-t}t^{\beta_1-1}}{\Gamma(\beta_1)}\ ;\ t\geq 0 \quad , \quad g_2(t) = \frac{e^{-t}t^{\beta_2-1}}{\Gamma(\beta_2)};\ t\geq 0$$

and $$h(t) = \frac{e^{-t}t^{\lambda-1}}{\Gamma(\lambda)}\ ;\ t\geq 0$$

Then the changed values of the results (3-10) and (18-22) are as follows—

$$p_{13} = (1+\alpha'_2)^{-\lambda} \ , \quad p_{17}^{(4)} = 1-(1+\alpha'_2)^{-\lambda}$$

$$p_{25} = (1+\alpha'_1)^{-\lambda} \ , \quad p_{28}^{(6)} = 1-(1+\alpha'_1)^{-\lambda}$$

$$p_{30} = (1+\alpha'_2)^{-\beta_1} \ , \quad p_{35}^{(7)} = 1-(1+\alpha'_2)^{-\beta_1}$$

$$p_{50} = (1+\alpha'_1)^{-\beta_2} \ , \quad p_{53}^{(8)} = 1-(1+\alpha'_1)^{-\beta_2}$$

$$\psi_1 = \frac{1}{\alpha'_2}[1-(1+\alpha'_2)^{-\lambda}] \ , \quad \psi_2 = \frac{1}{\alpha'_1}[1-(1+\alpha'_1)^{-\lambda}]$$

$$\psi_3 = \frac{1}{\alpha'_2}[1-(1+\alpha'_2)^{-\beta_1}] \ , \quad \psi_5 = \frac{1}{\alpha'_1}[1-(1+\alpha'_1)^{-\beta_2}]$$

$$\theta_1 = \beta_1 \ , \quad \theta_2 = \beta_2 \quad \text{and} \quad \phi = \lambda$$

8. Graphical representation and conclusions

The curves for MTSF and profit function are shown for the two particular cases-I and II in respect of different parameters. **In case-I, when repair time and administrative delay time follow Lindley distributions,** Fig. 2 and 3 depict the variations in MTSF and profit function w.r.t. the parameter α_1 for three different values of λ (= 0·5, 0·7 and 0·9) and two different values of β_1 (= 0·7 and 0·9) when the other parameters are kept fix as $\alpha_2 = 0{\cdot}05$, $\alpha'_1 = 0{\cdot}8$, $\alpha'_2 = 0{\cdot}1$ and $\beta_2 = 0{\cdot}7$. We may clearly observe from Fig. 2 that MTSF decreases uniformly as the value of α_1 increases. It is also revealed that MTSF increases with the increase in the value of λ as well as β_1. Similarly, Fig. 3 reveals the variations in profit with respect to α_1 for varying values of λ (= 0·5, 0·7 and 0·9) and β_1 (= 0·7 and 0·9) when the other parameters are taken as K_0 = 120, K_1 = 80, K_2 = 70, K_3 = 130, K_4 = 110 and K_5 = 80 in addition of above α_2, α'_1, α'_2, β_2. From Fig. 3 we observe that net expected profit decreases uniformly as α_1 increases but as compared to MTSF the reverse trends have been observed from smooth curves in respect of λ and same trend have been observed from dotted curves in respect of β_1. Also, it is obvious from dotted curves that the system is profitable only if α_1 is less than 0·675. 0·58 and 0·535 respectively for λ = 0·5, 0·7 and 0·9 for fixed value of β_1 = 0·9. Further, from smooth curves we conclude that system is profitable only if α_1 is less than 0·425, 0·393 and 0·375 respectively for λ = 0·5, 0·7 and 0·9 for fixed value of β_1 = 0·7.

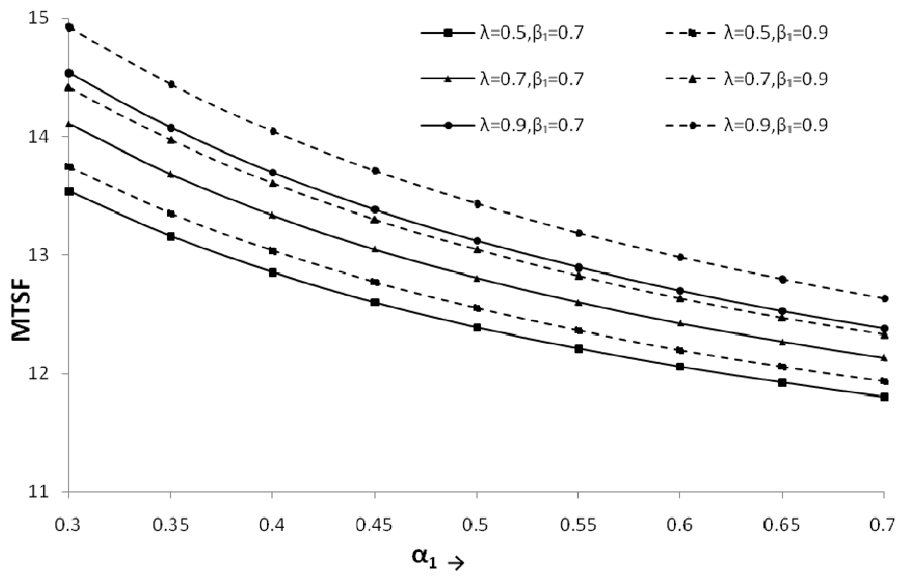

Fig.2

Behaviour of MTSF for case-1 with respect to α_1, λ and β_1

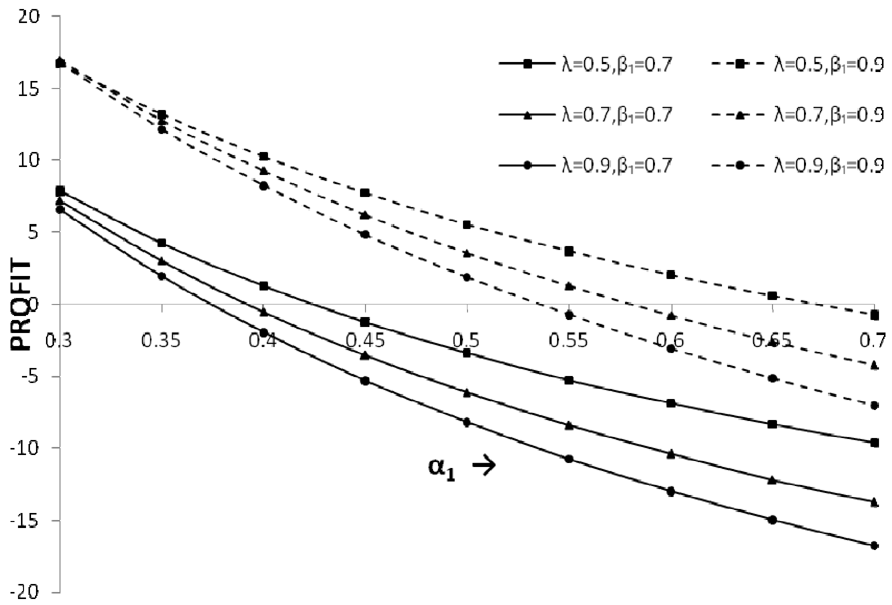

Fig. 3

Behaviour of Profit for case-1 with respect to α_1, λ and β_1

In case-II, when repair time and administrative delay time follow gamma distributions, Fig. 4 and 5 depict the variations in MTSF and profit function w.r.t. the parameter α_1 for three different values of λ and β_1. We may clearly observe from Fig. 4 and Fig. 5 that both MTSF and net expected profit decreases with the increase in the values of α_1. More so, MTSF decreases with the increase in the value of λ and β_1 but in case of profit it increases with the increase in λ and decreases with the increase in the values of β_1. Further, from Fig. 5 one can observe from smooth curves that system is profitable only if α_1 is less than 0·675. 0·625 and 0·58 respectively for λ = 0·9, 0·7 and 0·5 when β_1 is taken as 0·7. Similarly, from dotted curves, it is revealed that the system is profitable if α_1 is less than 0·54, 0·51 and 0·48 respectively for λ = 0·9, 0·7 and 0·5 when β_1 is fixed as 0·9.

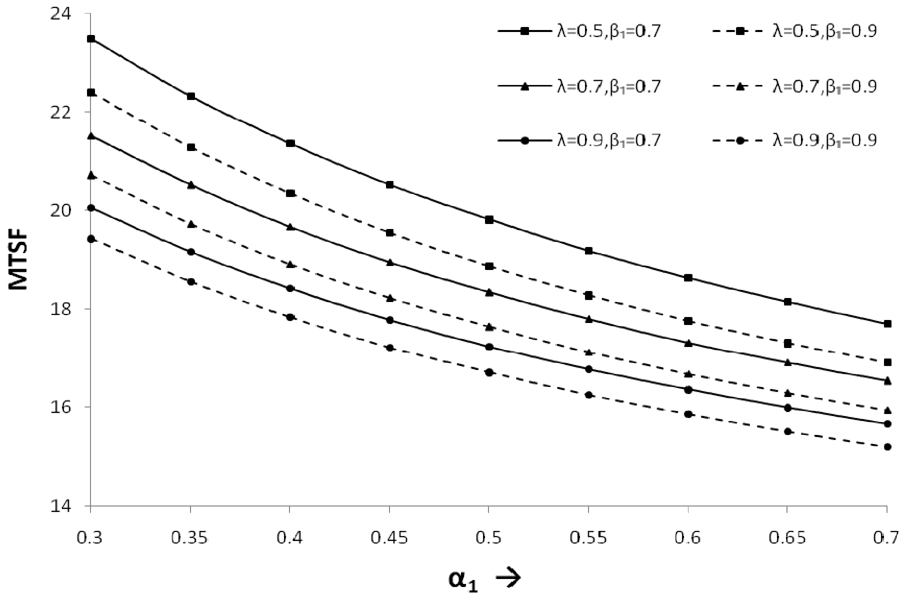

Fig.4

Behaviour of MTSF for case-2 with respect to α_1, λ and β_1

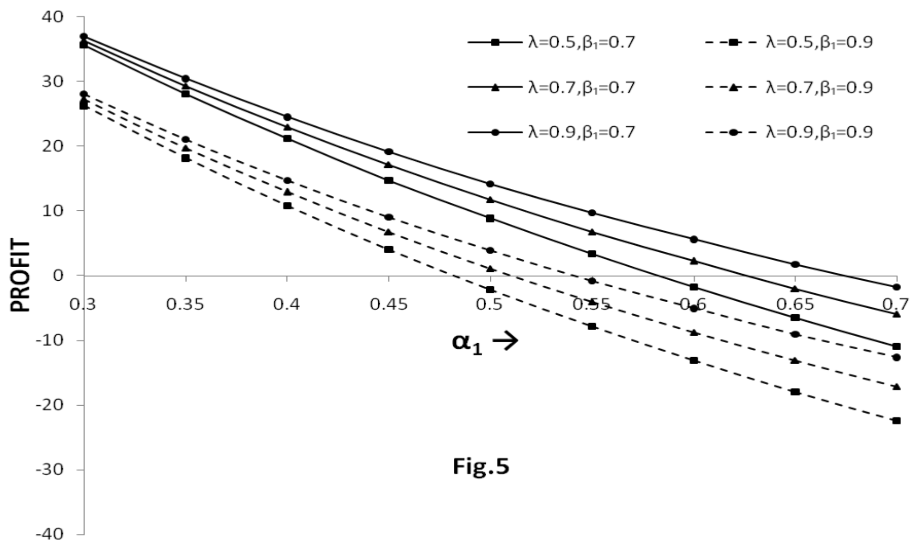

Fig.5

Behaviour of Profit for case-2 with respect to α_1, λ and β_1

Thus from the system model one can obtain the measures the system effectiveness for various continuous distributions of administrative delay time and repair times of unit-1 and unit-2. The bonds of any parameter can be evaluated for fixed values of other parameters to get non-negative profit. Moreso, one can also obtain the upper bond of any parameter (in case the curve is of decreasing nature w.r.t. this parameter) to achieve at least any specific value of MTSF and the lower bond of any parameter (in case the curve is of increasing nature w.r.t. this parameter) to achieve at least any particular value of MTSF. In view of above, the model has wide applicability for a large number of repair times and administrative delay time distributions to make various important economic decisions about the different parameters.

References

1. Chib, R., Joorel J.P.S. and Sharma, V. (2014). Profit analysis of a two-unit priority system with two-types of repair facility and proviso of rest, Int. J. of Statistics and Relaiability Engineering, 1(1), p. 69-81.
2. El-Said, K.M. and El-Sherbeny, M.S. (2010). Stochastic analysis of a two-unit cold standby system with two stage repair and waiting time, Sankhya; The Indian Journal of Statistics, 72-B, p. 01-10.
3. Gopalan M.N., Ramesh, T.K and Kumar Krishana (1986). Analysis of one server two unit parallel systems subject to degradation, Microelectronics Reliability, 26, p. 657-664.
4. Gupta, R and Goel, L.R. (1990). Cost-benefit analysis of two-unit parallel system with administrative delay in repair, Int. J. of System Science, 21, p. 1369-1379.
5. Jaiswal, N.K. and Krishna J.V. (1980). Analysis of two dissimilar unit standby redundant systems with administrative delay in repair, Int. J. of System Science, 11, p. 495-511.
6. Mogha, A.K and Gupta, A.K. (2002). A Two priority unit warm standby system model with preparation for repair, Aligarh J. of Statistics, 22, p. 73-90.
7. Mokaddis, G.S. and Tawfek, M.L. (1996). Some characteristics of a two-dissimilar unit cold standby redundant system with three modes, Microelectronics Reliability, 36(4).
8. Murari, K and Muruythachalam, C. (1981). Two-unit parallel system with period of working and rest, IEEE Transactions on Reliability, R-30(1), p. 187-190.
9. Nakagawa, T and Osaki, S. (1975). Stochastic behaviour of tow-unit parallel redundant system with preventive maintenance, Microelectronics Reliability, 14, p. 457-461.
10. Sridharan, V. And Mohanavadiva, P. (1998). Stochastic behaviour of a two-unit standby system with two types of repairman and patience time, Math. Comput. Modelling, 28, p. 63-71.