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FUZZY ANALYSIS OF MACHINE REPAIR PROBLEM WITH SWITCHING FAILURE AND REBOOT

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Abstract

Multi-component machining systems are being used in every sphere of engineering sector such as job shops, flow lines, communication system, computer system, etc. This paper presents fuzzy analysis of availability characteristics of machining system comprising of multi-active units and multi-standby units. The Markov machine repair model has been developed by incorporating the concepts of switching failure and reboot. The life times of identical active units and identical standby units follow the fuzzified exponential distribution. The time-to-repair of failed unit is also governed by the fuzzified exponential distribution. The automatic switching of standby unit to replace the failed units may not be perfect in many realistic scenarios as we assume the switching failure probability$q$. The system may reboot itself automatically if the active unit fails and available standby unit is not able to replace the failed unit perfectly. We employ the parametric non-linear program with α-cut approach to establish the membership function of availability of the system and availability of both standbys. A numerical example is also provided to validate the suggested approach which facilitates more useful information for the designers and practitioners to examine general repairable system more accurately.

Key Words: Availability, fuzzy set, membership function, machine repair problem, cold standby, switching failure, reboot delay.

1. Introduction

Machines which are an integral part of human kind in all sphere of life are prone to failure. The availability of machining system is great concern in many service systems and industrial organizations including production system and manufacturing systems, communication and computer systems, power plants, and other contexts. For just-in-time systems, it is very essential to maintain required level of reliability and/or availability of machining system. For improving the grade-of-service and availability of the system, many strategic, tactical and operational decisions have been adapted. The facility of standby units is helpful in maintaining the desired operating quality and high level of reliability and availability. With the advancement of technology, automatic switching of failed operating unit by standby unit takes place. But switching of standby unit may not be perfect which directly degrades the performance of the machining system. When a failed unit is not detected, located and recovered, it needs the time to be found and cleared. This can be done by rebooting the system and the time elapsed is termed as reboot delay. The switching failure and reboot delay are realistic key factors which directly degrade the system performance and availability measures. Such concepts should be taken into consideration while evaluating the performance indices of a machining system. The standby support is essential to achieve the high reliability/availability of any system operating in machining environment. A standby unit is termed as 'cold standby' if its failure rate is zero, 'warm standby' if its failure rate is less than that of an active unit and 'hot standby' if failure rate standby unit is equal to failure rate of an active unit. In this paper, we study a multi-component repairable machining system having operating and cold standby units by incorporating the switching failure and reboot delay concepts.

In traditional queueing models, the random variables denoting the time-to-failure and time-to-repair follow some standard/general probability distribution and require precise data. However, in many realistic queueing scenarios, the statistical information about these random variables is not precise due to uncontrolled factors. Due to subjective in nature, the parameters associated with the life time and repair time distributions can be interpreted only in linguistic terms like fast, moderate or slow. Fuzzy queueing models which deal with linguistic and imprecise congestion problems are more practical and realistic alternatives than commonly used classical models based on crisp parameter values. If the usual crisp or conventional finite population queueing models can be extended by using fuzzy logic, such models would have even wider applications in machining systems in different contexts. In the present study, we suggest parametric non-linear program to transform finite fuzzy queueing model of machine repair problem to conventional finite queue by applying the$\boldsymbol{\alpha}$-cut approach on membership function of availability parameters.

The application of our model can be realizedin security system where multi active devicese.g. camera, invigilation system, alarm, fire extinguishers, weapons, explosives, medical first aid, computersalong with standby devices are controlled by the robotic system for switching in place of failed device immediately. Due to some rare fault, this switching may not be perfect and whole system may go for rebooting processto overcome present situation as early as possible. The availability of such an alert system is the most important concern for any organization. This work differs from previous findings since we consider switching failure of cold standby and reboot delay simultaneously. The fuzzified time-to-failure and time-to-repair are considered to develop the finite capacity queueing model with multi active and standby units. To transform from crisp to fuzzy environment, the parametric non-linear program is employed so as to analyze the machine repair problem.

Aiming at the goal of deriving the membership functions of the performance measures, this article is organized as follows. Section 2 presents literature review where works of eminent researchers are outlined related to our problem. In section 3, we describe fuzzy machine repair problem by stating the assumptions and notations. In the same section, we develop Chapman-Kolmogrov differential equations for governing the model. In section 4, availability analysis is done by incorporating the availability of the system and availability of both standbys. In sections 5 and 6, parametric non-linear program based on Zadeh's extension principle is proposed to compute the$\boldsymbol{\alpha}$-cut from membership function of fuzzy performance measures. Numerical illustration and sensitivity analysis are given in section 7 to give more insight for our proposed model. Finally, conclusion and future scope are given in last section 8.

2. Literature Review

In literature, many studies have appeared on the machine repair problems and its reliability analysis [1]-[5]. Recently,[6] determined optimal number of standby units by successive over relaxation method to evaluate the minimum cost which depends on the state probabilities in multi-component machining system having K-type of standby units and operating under N policy.

Switching failure of standby system and reboot delay fascinated many queueing theorists in last decade[7]-[10].Introducing the supplementary variable corresponding to remaining repair time and using recursive approach, [11] developed reliability characteristics of multi-component repairable redundant system with coverage factor and reboot delay. Recently, [12] presented steady-state availability characteristics and queueingindices for machine repair problem with common cause failure and switching failure. [13], [14] presented availability analysis of machine repair problem with multi-type standby system under switching failure and reboot delay in crisp environment.

To deal with imprecise information in making decision [15], [16] introduced the concept of fuzziness. Today, fuzzy set theory is well-known method for modelling imprecision or uncertainty arising from mental phenomena. Text on fuzzy was enriched by many scholarsnamely[17], [18] and many more. [19]-[21] had done pioneer works in developing the theory in Fuzzy set and logic in their articles. Specifically, fuzzy queues have been discussed by several researchers to give broad insight in realistic models [22]-[30].[31] constructed the membership functions of the mean time to failure and availability of redundant repairable system consisting primary and standby units with fuzzified exponentially distributed failure and repair time.

With parametric non-linear program, the set of crisp intervals of system characteristics can be determined by $\boldsymbol{\alpha}$-cut approach. Observing reality of fuzzy set and logic in dealing with queueing problems, many researchers have used principle of fuzzy logic to analyze the queueing and reliability characteristics of the machine repair problem. [32]-[35]have done fuzzy analysis of machine repair problem to study the different characteristics and proposed different approaches to deal with reliability parameters also. [36]used$\boldsymbol{\alpha}$-cut approach to extract a family of conventional crisp intervals for mean time to failure and availability of the system. They have determined with set of parametric non-linear programs using their membership functions for repairable system having the active and standby units and unreliable service station with fuzzified exponentially distributed failure and repair times. [37]evaluated fuzzy reliability of repairable system with imperfect coverage, reboot and common cause failure.

3. System Description

In this paper, we propose parametric non-linear program to analyze availability analysis of machining system. The machining system consists of two active and two cold standby units under the invigilation of one reliable repairman. We assume the following assumptions to describe the present model:

- The mean time to failure of an active unit follows exponential distribution with parameter λ .
- The system consists of two cold standby units having zero mean failure rate. On failure of an active unit, automatic fault detection device replaces the failed unit with the available cold standby unit immediately. The replaced standby unit has same failure characteristics and working efficiency as that of the active unit.
- The switching time is negligible but automatic switching is not perfect as such there is significant probability q of switching failure. If the switching of cold standby unit is not successful, system tries switching with another standby unit, if available.
- The failed unit is immediately sent to the repair shop where single reliable repairman starts to provide repair immediately if available, otherwise failed units join the queue to wait for the repairman to become free; the repairman follows First Come First Serve service discipline.
- The mean time to repair of failed unitis exponentially distributed with rate μ. The repaired unit is as good as a new one or one before failure and joins the pool of either an active unit or a standby unit as per the system requirement.
- The reboot delay for a system for replacement with standby on failure of an active unit is exponentially distributed with mean time $1/\gamma$; the mean time of reboot delay is assumed to be too small for any other event to occur.
- All events like failure of unit, repair/replacement of a unit, reboot are statistically independent of the state of the others.

In order to develop Chapman-Kolmogrov differential difference equations of above defined Markov model of the repairable machining system, we define the following notations for state probabilities.

$P_{ij}(t)$ = Probability that there are i (1 or 2) active unit(s) and j (0,1 or 2) cold standby unit(s) in the system at time t

Using above defined assumptions and notations, the transition state diagram of the present stochastic model is shown in fig. 1.

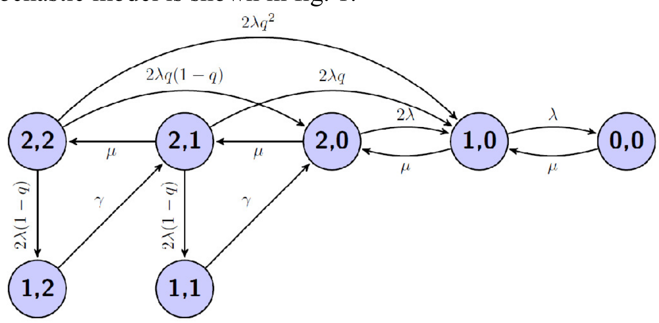

Figure 1: State transition diagram

Using the law of conserving the rate flows as depicted in Figure 1, following are the governing Chapman-Kolmogrov steady-state differential difference equations:

$$2\lambda P_{22} = \mu P_{21} \tag{1}$$

$$(2\lambda+\mu)P_{21}=\mu P_{20}+\gamma P_{12} \quad (2)$$

$$(2\lambda+\mu)P_{20}=2\lambda q(1-q)P_{22}+\mu P_{10}+\gamma P_{11} \quad (3)$$

$$(\lambda+\mu)P_{10}=2\lambda q^{2}P_{22}+2\lambda qP_{21}+2\lambda P_{20}+\mu P_{00} \quad (4)$$

$$\mu P_{00}=\lambda P_{10} \quad (5)$$

$$\gamma P_{12}=2\lambda(1-q)P_{22} \quad (6)$$

$$\gamma P_{11}=2\lambda(1-q)P_{21} \quad (7)$$

with normalizing condition

$$\sum_{i=1}^{2}\sum_{j=0}^{2}P_{ij}=1 \quad (8)$$

4. Availability Analysis

Reliability measures are of vital importance to predict the performance measures of machining system. In this section, we analyze availability of the system and availability of both standbys.For the brevity of notations, we denote

$$D=\mu^{4}\gamma+((2\gamma+(2\gamma(1+q)q+2(1-q)\mu)\mu^{3}+((4\gamma+2\gamma(q+4)q+4(1-q)\mu)\mu^{2}+ ((8\gamma(q+1)\mu+8\gamma\lambda)\lambda)\lambda)\lambda \quad (9)$$

The system of linear equations (1)-(7) with normalizing condition is solved using Gauss-elimination method to yield the following steady-state probabilities as follows:

$$P_{22}=\frac{\mu^{4}\gamma}{D} \quad (10)$$

$$P_{21}=\frac{2\lambda\gamma\mu^{3}}{D} \quad (11)$$

$$P_{20}=\frac{2\lambda\mu^{2}\gamma(2\lambda+\mu q)}{D} \quad (12)$$

$$P_{10}=\frac{2\lambda\mu\gamma(q^{2}\mu^{2}+4(\mu q+\lambda)\lambda)}{D} \quad (13)$$

$$P_{00}=\frac{2\lambda^{2}\gamma(q^{2}\mu^{2}+4(\mu q+\lambda)\lambda)}{D} \quad (14)$$

$$P_{12}=\frac{2\lambda\mu^{4}(1-q)}{D} \quad (15)$$

$$P_{11}=\frac{4\lambda^{2}\mu^{3}(1-q)}{D} \quad (16)$$

Since state 00 is down state of the system, the availability of the system is defined as

$$A=1-P_{00}$$

$$=\frac{\mu(\mu^{3}\gamma+((2\gamma+(2\gamma(1+q)q+2(1-q)\mu)\mu^{2}+((8q\gamma+4\gamma+4(1-q)\mu)\mu+8\gamma\lambda)\lambda)\lambda)}{D} \quad (17)$$

The availability of the both standbys is defined and obtained using

$$A_{22} = P_{00} = \frac{\mu^4 \gamma}{D} \tag{18}$$

5. Fuzzy Analysis

To extend the applicability of the repairable machining system in more realistic scenario, we consider the fuzzy specification of system parameters. Suppose the failure rate of an active unit (λ), reboot delay (γ), repair rate of failed units (μ) are approximated by fuzzy sets λ^*, γ^* and μ^* respectively obtained from respective crisp universal sets X, Y and Z.

Let $n_{\lambda^*}(x)$, $n_{\gamma^*}(y)$ and $n_{\mu^*}(z)$ be the membership functions of λ^*, γ^* and μ^* respectively. Then

$$\lambda^* = \left\{ \left(x, n_{\lambda^*}(x) \right) \middle| x \in X \right\} \tag{19}$$

$$\gamma^* = \left\{ \left(y, n_{\gamma^*}(y) \right) \middle| y \in Y \right\} \tag{20}$$

$$\mu^* = \left\{ \left(z, n_{\mu^*}(z) \right) \middle| z \in Z \right\} \tag{21}$$

Let $A(x, y, z)$ and $A_{22}(x, y, z)$ denote the availability of the system and availability of both standbys. Since λ^*, γ^* and μ^* are fuzzy numbers, $A^*\left(\lambda^*, \gamma^*, \mu^*\right)$ and $A^*_{22}\left(\lambda^*, \gamma^*, \mu^*\right)$ are also fuzzy numbers.Using Zadeh's Principle [16], the membership functions for theseavailability characteristic are denoted by

$$n_{A^*\left(\lambda^*, \gamma^*, \mu^*\right)}(r) = \sup_{\substack{x \in X, y \in Y, z \in Z \\ x, y, z > 0}} \min \left\{ n_{\lambda^*}(x), n_{\gamma^*}(y), n_{\mu^*}(z) \middle| r = A(x, y, z) \right\} \tag{22}$$

$$n_{A^*_{22}\left(\lambda^*, \gamma^*, \mu^*\right)}(s) = \sup_{\substack{x \in X, y \in Y, z \in Z \\ x, y, z > 0}} \min \left\{ n_{\lambda^*}(x), n_{\gamma^*}(y), n_{\mu^*}(z) \middle| s = A_{22}(x, y, z) \right\} \tag{23}$$

where $A(x, y, z)$ and $A_{22}(x, y, z)$ are given by (17) and (18) respectively by substituting x, y and z for λ, γ and μ respectively.

6. Parametric Non Linear Programs

In this section we employ the parametric non-linear program, a mathematical program, to obtain crisp value of availability characteristics established in previous section. To understand the form of membership function, we make use of Zadeh's α -cut approach.We define the α -cuts for λ^*, γ^* and μ^* as crisp intervals as follows:

$$\lambda(\alpha) = \left[x_\alpha^L, x_\alpha^U \right] = \left[\min_{x \in X} \left\{ x \middle| n_{\lambda^*}(x) \geq \alpha \right\}, \max_{x \in X} \left\{ x \middle| n_{\lambda^*}(x) \geq \alpha \right\} \right] \tag{24}$$

$$\gamma(\alpha) = \left[y_\alpha^L, y_\alpha^U \right] = \left[\min_{y \in Y} \left\{ y \middle| n_{\gamma^*}(y) \geq \alpha \right\}, \max_{y \in Y} \left\{ y \middle| n_{\gamma^*}(y) \geq \alpha \right\} \right] \tag{25}$$

$$\mu(\alpha) = \left[z_\alpha^L, z_\alpha^U \right] = \left[\min_{z \in Z} \left\{ z \middle| n_{\mu^*}(z) \geq \alpha \right\}, \max_{z \in Z} \left\{ z \middle| n_{\lambda^*}(z) \geq \alpha \right\} \right] \tag{26}$$

The constant rates λ, γ and μ are extended as intervals when the membership functions are no less than a given possibility level α. Thus the bounds of these intervals can be mapped as a function of α. Since membership functions $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s)$ defined in the previous section are parameterised by α, we can use α-cut of A^* and A^*_{22} to obtain their membership function, respectively. Therefore, to find the membership function $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s)$, it is sufficient to find left and right shape functions of $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s)$ which is equivalent to find corresponding lower bounds A^L_α & $A_{22}{}^L_\alpha$ and upper bounds A^U_α & $A_{22}{}^U_\alpha$ of the α-cuts of A^* and A^*_{22} which are given by:

$$A^L_\alpha = \min_{\substack{x\in X, y\in Y, z\in Z \\ x,y,z>0}} A^* \tag{27}$$

$$A^U_\alpha = \max_{\substack{x\in X, y\in Y, z\in Z \\ x,y,z>0}} A^* \tag{28}$$

$$A_{22}{}^L_\alpha = \min_{\substack{x\in X, y\in Y, z\in Z \\ x,y,z>0}} A^*_{22} \tag{29}$$

$$A_{22}{}^U_\alpha = \max_{\substack{x\in X, y\in Y, z\in Z \\ x,y,z>0}} A^*_{22} \tag{30}$$

where $x^L_\alpha \le x \le x^U_\alpha$, $y^L_\alpha \le y \le y^U_\alpha$ and $z^L_\alpha \le z \le z^U_\alpha$. The crisps intervals $\left[A^L_\alpha, A^U_\alpha\right]$ and $\left[A_{22}{}^L_\alpha, A_{22}{}^U_\alpha\right]$ obtained in (27)-(30) represent the α-cut of A^* and A^*_{22}. Following [17], [18] and convexity properties of A^* and A^*_{22}, we have $A^L_{\alpha_1} \ge A^L_{\alpha_2}$, $A^U_{\alpha_1} \le A^U_{\alpha_2}$, $A_{22}{}^L_{\alpha_1} \ge A_{22}{}^L_{\alpha_2}$ and $A_{22}{}^U_{\alpha_1} \le A_{22}{}^U_{\alpha_2}$ for $0 < \alpha_1 \le \alpha_2 < 1$. Thus A^L_α and $A_{22}{}^L_\alpha$ increase and A^U_α and $A_{22}{}^U_\alpha$ decrease as α increases. So, finally membership functions $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s)$ can be obtained from (22)-(23). If both A^L_α & A^U_α and $A_{22}{}^L_\alpha$ & $A_{22}{}^U_\alpha$ are invertible with respect to α, then the explicit expressions for left shape functions $LA(r) = \left[A^L_\alpha\right]^{-1}$ & $LA_{22}(s) = \left[A_{22}{}^L_\alpha\right]^{-1}$ and a right shape functions $RA(r) = \left[A^U_\alpha\right]^{-1}$ & $RA_{22}(s) = \left[A_{22}{}^U_\alpha\right]^{-1}$ can be derived, consequently membership functions of $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s)$ are constructed as

$$n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)=\begin{cases} LA(r); A_{\alpha=0}^{L} \le r \le A_{\alpha=1}^{L} \\ 1; A_{\alpha=1}^{L} \le r \le A_{\alpha=1}^{U} \\ RA(r); A_{\alpha=1}^{U} \le r \le A_{\alpha=0}^{U} \end{cases} \tag{31}$$

$$n_{A_{22}^*(\lambda^*,\gamma^*,\mu^*)}(s)=\begin{cases} LA_{22}(s); A_{22}{}_{\alpha=0}^{L} \le s \le A_{22}{}_{\alpha=1}^{L} \\ 1; A_{22}{}_{\alpha=1}^{L} \le s \le A_{22}{}_{\alpha=1}^{U} \\ RA_{22}(s); A_{22}{}_{\alpha=1}^{U} \le s \le A_{22}{}_{\alpha=0}^{U} \end{cases} \tag{32}$$

In a complex system, it is very difficult to derive explicit expressions of A_α^L & A_α^U and $A_{22}{}_\alpha^L$ & $A_{22}{}_\alpha^U$ as such we cannot derive closed form membership functions $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A_{22}^*(\lambda^*,\gamma^*,\mu^*)}(s)$ of A^* and A^*_{22}. For approximating the shape of $LA(r)$, $RA(r)$, $LA_{22}(s)$ and $RA_{22}(s)$, numerical solution for A_α^L & A_α^U and $A_{22}{}_\alpha^L$ & $A_{22}{}_\alpha^U$ at different possibility level can be collected i.e. the set of intervals $\{[A_\alpha^L, A_\alpha^U] \alpha \in [0,1]\}$ and $\{[A_{22}{}_\alpha^L, A_{22}{}_\alpha^U] \alpha \in [0,1]\}$ show shape of $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A_{22}^*(\lambda^*,\gamma^*,\mu^*)}(s)$ respectively although exact functions are not known explicitly. Membership function of availability characteristics preserves all fuzziness of the governing rates finally.

Usually system designers require single crisp value of availability characteristic rather than the fuzzy set for effective and appropriate analysis. For the same purpose, fuzzy values are defuzzified for the availability characteristic using Yager's ranking index method [Yager (1986)].

7. Numerical Results

Suppose in a security system, there are two active units with two cold standby units for maintaining safety in an organization. With experience and past record, it observed that the failure rate of an active unit λ, reboot delay γ, repair rate of the failed unit μ are trapezoidal fuzzy numbers given as $\lambda^* = [1,2,3,4]$, $\gamma^* = [20,25,30,35]$, $\mu^* = [5,6,7,8]$ and switching failure probability $q = 0.5$. Thus from (24)-(26), α -cuts for λ^*, γ^* and μ^* as crisp intervals are computed as follows:

$$\lambda(\alpha) = [x_\alpha^L, x_\alpha^U] = [1+\alpha, 4-\alpha] \tag{33}$$

$$\gamma(\alpha) = [y_\alpha^L, y_\alpha^U] = [20+5\alpha, 35-5\alpha] \tag{34}$$

$$\mu(\alpha) = [z_\alpha^L, z_\alpha^U] = [5+\alpha, 8-\alpha] \tag{35}$$

Next, we find heuristically that the availability of the system attains its minimum value for $x = x_\alpha^U$, $y = y_\alpha^U$ and $z = z_\alpha^L$ and maximum value for $x = x_\alpha^L$, $y = y_\alpha^L$ and $z = z_\alpha^U$. Similarly availability of both standby attains its minimum value for

$x = x_\alpha^U$, $y = y_\alpha^L$ and $z = z_\alpha^L$ and maximum value for $x = x_\alpha^L$, $y = y_\alpha^U$ and $z = z_\alpha^U$. From equations (27) and (28), the minimum and maximum values of the availability of the system are given by

$$A_\alpha^L = \frac{(\alpha+5)(-38830+(15825+(-2973+(283-9\alpha)\alpha)\alpha)\alpha)}{-276470+(116735+(-26865+(3407+(-197+6\alpha)\alpha)\alpha)\alpha)\alpha} \quad (36)$$

$$A_\alpha^U = \frac{(8-\alpha)(11200+(4656+(912+(175+9\alpha)\alpha)\alpha)\alpha)}{90560+(28688+(5220+(1583+(107+6\alpha)\alpha)\alpha)\alpha)\alpha} \quad (37)$$

Similarly from equations (29) and (30), the minimum and maximum values of availability of both standbys are given by

$$A_{22\alpha}^{L} = \frac{10(\alpha+5)^4(\alpha+4)}{479520+(-15320+(-7495+(4704+(-379+22\alpha)\alpha)\alpha)\alpha)\alpha} \quad (38)$$

$$A_{22\alpha}^{U} = \frac{10(\alpha-8)^4(\alpha-7)}{-467760+(34696+(-20315+(2136+(49+22\alpha)\alpha)\alpha)\alpha)\alpha} \quad (39)$$

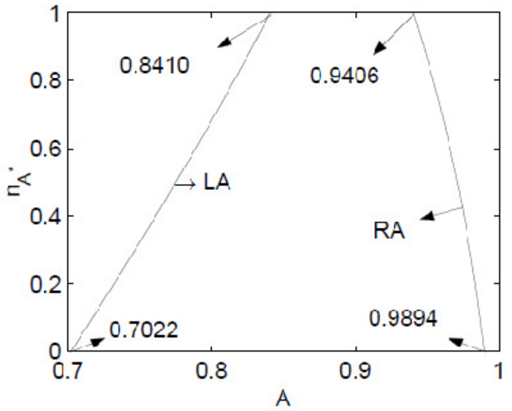

Figure 2: Membership function for fuzzy availability of the system

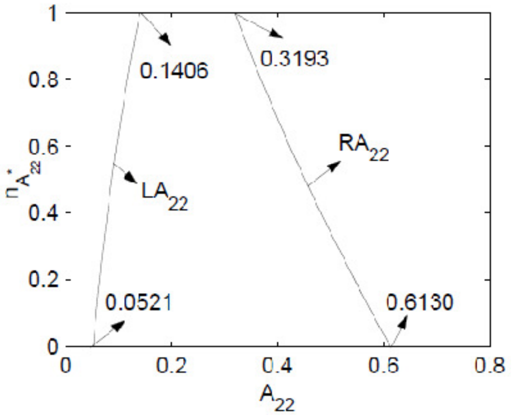

Figure 3: Membership function for fuzzy availability of both standbys

Expressions in equation (33)-(36) are highly non-linear in nature. The explicit expressions for $n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r)$ and $n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s)$ are not possible. The graphical representation of equations (31) and (32) are depicted in figures 2 and 3, respectively. From these figures, it is clear that

$$n_{A^*(\lambda^*,\gamma^*,\mu^*)}(r) = \begin{cases} LA(r); 0.7022 \le r \le 0.8410 \\ 1; 0.8410 \le r \le 0.9406 \\ RA(r); 0.9406 \le r \le 0.9894 \end{cases} \quad (40)$$

$$n_{A^*_{22}(\lambda^*,\gamma^*,\mu^*)}(s) = \begin{cases} LA_{22}(s); 0.0521 \le s \le 0.1406 \\ 1; 0.1406 \le s \le 0.3193 \\ RA_{22}(s); 0.3193 \le s \le 0.6130 \end{cases} \quad (41)$$

These figures also depicttwo ample results for fuzzy availability of the system A^* and fuzzy availability of the both standbys A^*_{22} . Firstly, the support of A^* ranges from 0.7022 to 0.9894; this infers that the availability of the system is fuzzy in nature and its value can't fall below 0.7022 or exceed 0.9894 under present state-of-art of the system and secondly, most possible value of availability of the system ranges from 0.8410 to 0.9406 since this is the core of A^* (with height 1). Similarly, for availability of both standby, support of A^*_{22} ranges from 0.0521 to 0.6130 and core of A^*_{22} ranges from 0.1406 to 0.3193.

For more insight in our study, we conduct simulation to explore the sensitivity of some parameters. Numerical results are displayed via graphs. For trapezoidal fuzzy number of following fuzzy parameter $\lambda^* = [1,2,3,4]$, $\gamma^* = [20,25,30,35]$ and $\mu^* = [5,6,7,8]$, figure 4 reveals the membership function of availability characteristics with respect to switching failure probabilities $q = 0.01$, $q = 0.1$ and $q = 0.9$.

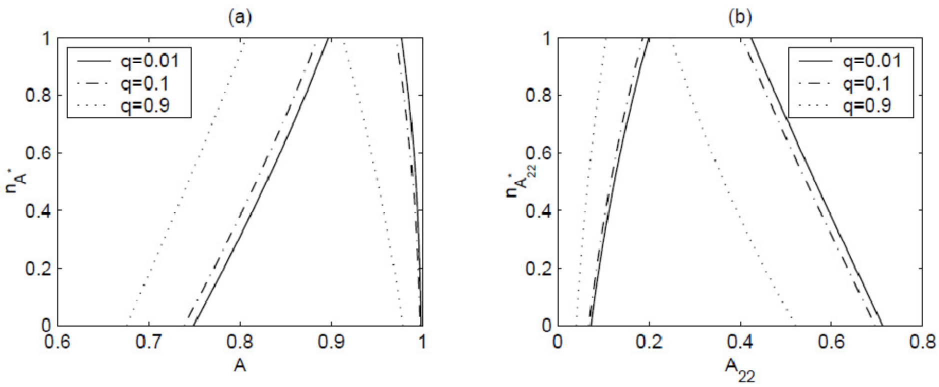

Figure 4: Membership function for fuzzy availability characteristics w.r.t. q

Figure 4 reveals that the range of possible values of A^* get narrower and expected possible value increases on decreasing the value of q. Expected possible value of A_{22}^* also increases on decreasing the value of q. These results show that switching failure probability should be minimised as much as possible to get more availability of the system and both standbys which are obvious.

For $q = 0.01$, $\gamma^* = [20,25,30,35]$ and $\mu^* = [5,6,7,8]$, in figure 5, we plot membership functions for fuzzy A^* and A_{22}^* for trapezoidal fuzzy number for failure rate of an active unit λ^* as $\lambda_1 = [1,2,3,4]$, $\lambda_2 = [2,3,4,5]$ and $\lambda_3 = [3,4,5,6]$.

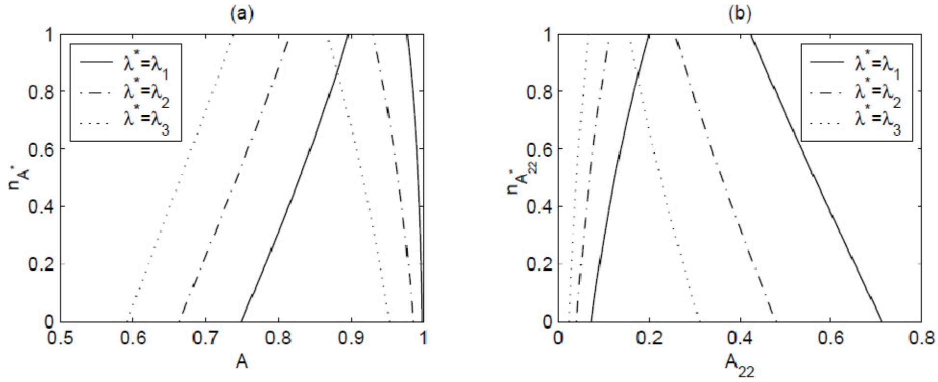

Figure 5: Membership function for fuzzy availability characteristics w.r.t. λ^*

From figure 5, it is clear that on increasing the fuzzy failure rate of an active unit, expected possible values of A^* and A_{22}^* decrease but the range of fuzzy number A^* gets broader and A_{22}^* gets narrow. It is recommended that the system designers should use some preventive measures of high grade to avoid frequent failure of an active unit.

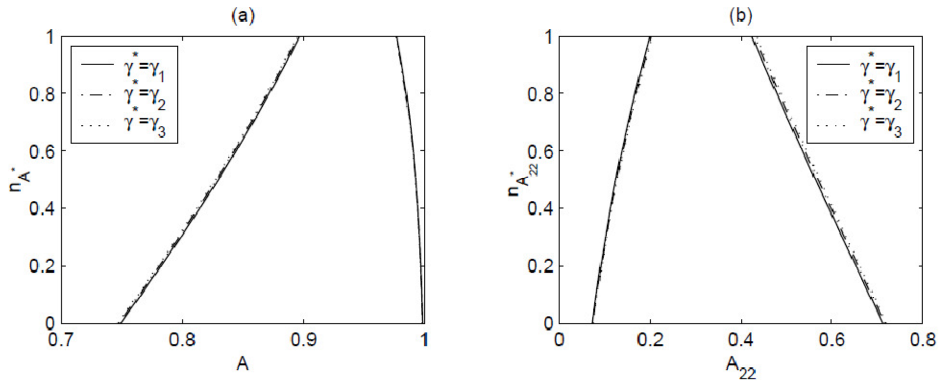

Figure 6: Membership function for fuzzy availability characteristics wrt γ^*

For given $q = 0.01$, $\lambda^* = [1,2,3,4]$ and $\mu^* = [5,6,7,8]$, in figure 6, we display the pattern of membership function for fuzzy number A^* and A^*_{22} for possible value of reboot delay γ^* as $\gamma_1 = [20,25,30,35]$, $\gamma_2 = [25,30,35,40]$ and $\gamma_3 = [30,35,40,45]$. It can be noticed from figure 6 that even for sufficiently high and different possible values of reboot delay, it does not alter availability characteristics much. The system designer must not pay much attention in automatic reboot policy. Sufficient high rate is enough for normal function of the system. For analysing the effect of service rate μ, we consider wide range of possible values of service rate. Assuming $q = 0.01$ and trapezoidal fuzzy number $\lambda^* = [1,2,3,4]$ and $\gamma^* = [20,25,30,35]$, we present membership functions of fuzzy number A^* and A^*_{22} in figure 7 for following trapezoidal numbers for μ^*: $\mu_1 = [5,6,7,8]$, $\mu_2 = [7,8,9,10]$, and $\mu_3 = [9,10,11,12]$.

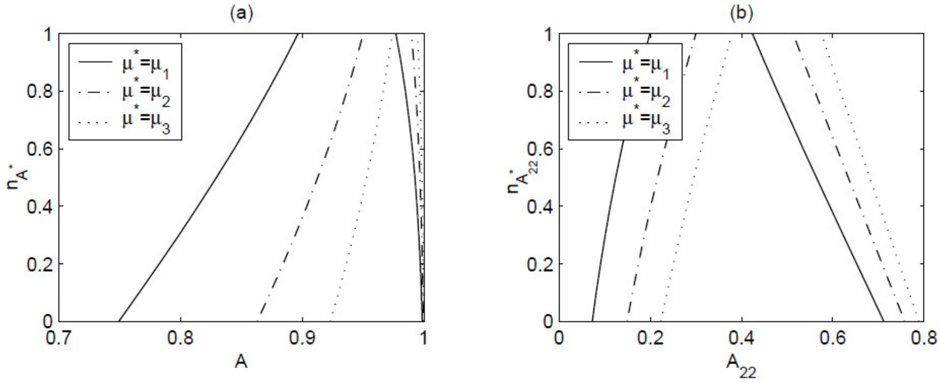

Figure 7: Membership function for fuzzy availability characteristics w.r.t. μ^*

Figure 7 reveals that for high possible values of service rate, the range of support$\left(A^*\right)$ gets narrower and expected possible values of availability characteristics A^* and A^*_{22} get increased. In such a case, a high skilled repairman must be installed with the machine repair system to get desired availability of the system. Based on numerical results, the following points should be taken into consideration:

- For present state of the art, it is impossible for the availability characteristic to fall below the left intercept in pessimistic condition or exceed the right intercept in optimistic condition on the characteristic axis.
- α -cut at $\alpha = 1$ contains the values which are most probable for availability characteristic in most likely condition.

8. Conclusions

In this investigation we have proposed parametric non-linear program to yield membership function of availability characteristics from α -cut of repairable machining system. We have evaluated system performance for realistic machine repair problem having two active units with support of two cold standbys. We have incorporated more realistic assumptions of switching failure and reboot delay which makes our model more closure to real time system in practical context also. We have suggested that how to convert fuzzified availability characteristics to crisp value using Yager's defuzzification principle. The numerical results provided may be helpful to the decision makers and industrial engineers to improve the availability and maintenance strategy of the concerned machine repair system operating under fuzzy environment.

In future we aim to extend our study for general time-to-failure and/or time-to-repair distribution. The model can be developed for unreliable repairman system for which work is in progress. We can also generalize the present model for M operating units and N standby units for machining system having provisioning of mixed standbys.

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