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A NEW PROCEDURE FOR ESTIMATION OF FINITE POPULATION VARIANCE USING AUXILIARY INFORMATION

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Abstract

This paper addresses the problem of estimating the population variance of variable of interest using information on an auxiliary variable in sample surveys. A new class of ratio-type estimators is proposed. In addition to some existing estimators, large number of consistent and inconsistent estimators has been identified as a member of the proposed class of estimators. The bias and mean square error of the proposed class of estimators are obtained to the first order of approximation. The minimum mean square error of the proposed class of estimators is also obtained. The proposed class of ratio-type estimators has been compared with the usual unbiased estimator and ratio-type estimators. An empirical study has been carried out to assess the performance of the proposed estimator.

Key Words: Auxiliary Variable, Study Variable, Bias, Mean Square Error, Simple Random Sampling.

AMS Classification: 62D05.

1. Introduction

In manufacturing industries and pharmaceutical laboratories sometimes researchers are interested in the variation of their products. To measure the variations within the values of the study variable *y*, the problem of estimating the population variance S_y^2 of the study variable *y* also received a considerable attention in survey sampling, see Jhajj et al. (2005). It is well known that the use of auxiliary information at the estimation stage improves the estimates of population parameters such as the population mean $\bar{Y}$ or total $Y(=\bar{Y})$, variance S_y^2 and coefficient of variation $C_y(=S_y/\bar{Y})$ etc. of the study variable *y*. Ratio , product and regression methods of estimation are good examples in this context. It is assumed that the population variance S_x^2 of the auxiliary variable *x* is known in advance. In this situation, several authors including Das and Tripathi (1978), Srivastava and Jhajj (1980), Isaki (1983), Singh et al. (1988), Prasad and Singh (1990, 1992), Biradar and Singh (1994, 1998), Upadhyaya and Singh (1999, 2001), Kadilar and Cingi (2006, 2007), Singh and Solanki (2013) and

Singh et al. (2013) etc. have paid their attention towards the estimation of population variance S_y^2.

Consider a finite population $U = \{U_1, U_2, .., U_i, .., U_N\}$ consisting of *N* units. Let *y* and *x* be the study and auxiliary variables with population means $\overline{Y}$ and $\overline{X}$ respectively. Let there be a sample of size *n* drawn from the population *U* using simple random sampling without replacement (*SRSWOR*). Let s_y^2 and s_x^2 be the sample variances with devisors (*n*-1) for variables *y* and *x*, which are unbiased estimators of the population variances S_y^2 and S_x^2 respectively. Let $C_y (= S_y / \overline{Y})$ and $C_x (= S_x / \overline{X})$ be the coefficients of variation of *y* and *x* respectively, and ρ the coefficient of correlation between *y* and *x*. We assume that all parameters of the auxiliary variable *x* are known, see, Gupta and Shabbir (2008, p.58).

Further in what follows, we shall use the following notations:

Q_1 : First (lower) quartile of the auxiliary variable *x*

Q_3 : Third (upper) quartile of the auxiliary variable *x*

$Q_r = (Q_3 - Q_1)$: Inter- quartile range of the auxiliary variable *x*

$Q_d = (Q_3 - Q_1)/2$: Semi- quartile range of the auxiliary variable *x*

$Q_r = (Q_3 + Q_1)/2$: Semi- quartile average of the auxiliary variable *x*

Let $e_0 = (s_y^2 - S_y^2)/S_y^2$ and $e_1 = (s_x^2 - S_x^2)/S_x^2$ be such that $E(e_0) = E(e_1) = 0$. Also ignoring finite population correction (fpc) term and to the first degree of approximation, we have

$E(e_0^2) = \gamma(\lambda_{40} - 1) = \gamma(\beta_2(y) - 1)$,

$E(e_1^2) = \gamma(\lambda_{04} - 1) = \gamma(\beta_2(x) - 1)$, $E(e_0 e_1) = \gamma(\lambda_{22} - 1)$,

where $\gamma = 1/n$, $\lambda_{pq} = \mu_{pq} / (\mu_{02}^{p/2} \mu_{20}^{q/2})$, $\mu_{pq} = N^{-1} \sum_{i-1}^{N} (y_i - \overline{Y})^p (x_i - \overline{X})^q$,

(*p*,*q*) being non negative integers, and $\lambda_{40} = \beta_2(y) = \mu_{40} / \mu_{20}^2$, $\lambda_{04} = \beta_2(x) = \mu_{04} / \mu_{02}^2$ are the coefficients of Kurtosis of *y* and *x* respectively.

Recall that the variance /*MSE* (ignoring finite population correction term) of the usual unbiased estimator

$$l_0 = s_y^2 = (n-1)^{-1} \sum_{i=1}^{n} (y_i - \overline{y})^2 , \quad (1)$$

is given by

$$V(l_0) = MSE(l_0) = \gamma S_y^4 (\lambda_{40} - 1), \quad (2)$$

Isaki (1983) suggested the following ratio estimator for population variance S_y^2 defined by

$$l_1 = s_y^2\left(S_x^2/s_x^2\right) \tag{3}$$

The bias and mean square error (*MSE*) of l_1 ignoring finite population correction (fpc) term are given as

$$B(l_1) = \gamma S_y^2(\lambda_{04}-1)(1-k), \tag{4}$$

$$MSE(l_1) = \gamma S_y^4\left[(\lambda_{40}-1)+(\lambda_{04}-1)(1-2k)\right], \tag{5}$$

where $k = (\lambda_{04}-1)^{-1}(\lambda_{22}-1)$.

The remaining part of the paper is organized as follows: In Section2, an improved class of ratio-type estimators of population variance S_y^2 has been suggested and expressions of its bias and mean square error to the first degree of approximation are obtained. Section 3 provides efficiency comparisons, while Section 4 has focused on empirical study of proposed class of estimators. Section 5 finished off the paper with final remarks.

2. The proposed class of estimators

We consider the following class of estimators of population variance S_y^2 as

$$l = s_y^2\left(\frac{aS_x^2+bs_x^2}{cs_x^2+dS_x^2}\right), \tag{6}$$

where (a,b,c,d) are suitably chosen scalars such that $l>0$. Scalars (a,b,c,d) may assume real values as well as parametric values such as C_y (coefficient of variation of *y*), C_x (coefficient of variation of *x*), ρ (correlation coefficient between *y* and *x*), $\beta_1(x)$ (coefficient of skewness of *x*), $\beta_2(x)$ (coefficient of kurtosis of *x*), etc. It is to be noted that for suitable choices of the scalars (a,b,c,d) , the proposed class of estimators *l* reduces to the set of some known consistent estimators of S_y^2 given in Table 1.

To obtain the bias and mean square error (*MSE*) of the proposed class of estimators *l*, we express *l* at (6) in terms of e_0 and e_1 as

$$\begin{aligned} l &= S_y^2(1+e_0)\left[\frac{aS_x^2+bS_x^2(1+e_1)}{cS_x^2(1+e_1)+dS_x^2}\right] \\ &= S_y^2(1+e_0)\left[\frac{a+b(1+e_1)}{c(1+e_1)+d}\right] \\ &= S_y^2(1+e_0)\left(\frac{a+b}{c+d}\right)\left(1+\frac{be_1}{a+b}\right)\left(1+\frac{ce_1}{c+d}\right)^{-1} \end{aligned}$$

$$= \left(\frac{a+b}{c+d}\right) S_y^2 (1+e_0)(1+\theta_1 e_1)(1+\theta_2 e_1)^{-1}$$
$$= AS_y^2 (1+e_0)(1+\theta_1 e_1)(1+\theta_2 e_1)^{-1}, \qquad (7)$$

where $A = (a+b)/(c+d)$, $\theta_1 = b/(a+b)$ and $\theta_2 = c/(c+d)$.

We assume that $|\theta_2 e_1| < 1$ so that $(1+\theta_2 e_1)^{-1}$ is expandable .Now expanding the right hand side of (7) and multiplying out, we have

$$l = AS_y^2[1+e_0+\theta_1 e_1+\theta_1 e_0 e_1-\theta_2 e_1-\theta_2 e_0 e_1-\theta_1\theta_2 e_1^2+\theta_2^2 e_1^2 \ldots]$$
$$= AS_y^2[1+e_0+(\theta_1-\theta_2)e_1+(\theta_1-\theta_2)e_0 e_1-\theta_2(\theta_1-\theta_2)e_1^2 \ldots], \qquad (8)$$

where $\theta = (\theta_1 - \theta_2)$.

Constants				**Estimator**
a	***b***	***c***	***d***	
1	1	1	1	$l_0 = s_y^2$ [The usual unbiased estimator]
1	0	1	0	$l_1 = l_R = s_y^2(S_x^2 / s_x^2)$ [Isaki (1983)]
$\frac{S_x^2 - C_x}{S_x^2}$	0	1	$-\frac{C_x}{S_x^2}$	$l_2 = s_y^2\left(\frac{S_x^2 - C_x}{s_x^2 - C_x}\right)$ [Kadilar and Cingi (2006)]
$\frac{S_x^2 - \beta_2(x)}{S_x^2}$	0	1	$-\frac{\beta_2(x)}{S_x^2}$	$l_3 = s_y^2\left(\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right)$ [Kadilar and Cingi (2006)]
$\frac{S_x^2\beta_2(x) - C_x}{S_x^2}$	0	$\beta_2(x)$	$-\frac{C_x}{S_x^2}$	$l_4 = s_y^2\left(\frac{S_x^2\beta_2(x) - C_x}{s_x^2\beta_2(x) - C_x}\right)$ [Kadilar and Cingi (2006)]
$\frac{S_x^2 C_x - \beta_2(x)}{S_x^2}$	0	C_x	$-\frac{\beta_2(x)}{S_x^2}$	$l_5 = s_y^2\left(\frac{S_x^2 C_x - \beta_2(x)}{s_x^2 C_x - \beta_2(x)}\right)$ [Kadilar and Cingi (2006)]
$\frac{S_x^2 + Q_1}{S_x^2}$	0	1	$\frac{Q_1}{S_x^2}$	$l_6 = s_y^2\left(\frac{S_x^2 + Q_1}{s_x^2 + Q_1}\right)$ [Subramani and Kumarapandiyan (2012)]
$\frac{S_x^2 + Q_3}{S_x^2}$	0	1	$\frac{Q_3}{S_x^2}$	$l_7 = s_y^2\left(\frac{S_x^2 + Q_3}{s_x^2 + Q_3}\right)$ [Subramani and Kumarapandiyan (2012)]
$\frac{S_x^2 + Q_r}{S_x^2}$	0	1	$\frac{Q_r}{S_x^2}$	$l_8 = s_y^2\left(\frac{S_x^2 + Q_r}{s_x^2 + Q_r}\right)$ [Subramani and Kumarapandiyan (2012)]

$\frac{S_x^2+Q_d}{S_x^2}$	0	1	$\frac{Q_d}{S_x^2}$	$l_9 = s_y^2\left(\frac{S_x^2+Q_d}{s_x^2+Q_d}\right)$ [Subramani and Kumarapandiyan (2012)]
$\frac{S_x^2+Q_a}{S_x^2}$	0	1	$\frac{Q_a}{S_x^2}$	$l_{10} = s_y^2\left(\frac{S_x^2+Q_a}{s_x^2+Q_a}\right)$ [Subramani and Kumarapandiyan (2012)]
$\frac{S_x^2\rho+Q_3}{S_x^2}$	0	ρ	$\frac{Q_3}{S_x^2}$	$l_{11} = s_y^2\left(\frac{S_x^2\rho+Q_3}{s_x^2\rho+Q_3}\right)$ [Khan and Shabbir (2013)]

Table 1: Some known consistent members of suggested class of estimators *l*.

Neglecting terms of e 's having power greater than two in(8), we have

$$l \cong AS_y^2[1+e_0+\theta\{e_1+e_0e_1-\theta_2e_1^2\}]$$

or

$$(l-S_y^2) \cong S_y^2[A\{1+e_0+\theta(e_1+e_0e_1-\theta_2e_1^2)\}-1]. \tag{9}$$

Taking expectation of both sides of (9), we get the bias of l, to the first degree of approximation as

$$B(l) = S_y^2[A\{1+\gamma\theta(\lambda_{04}-1)(k-\theta_2)\}-1]. \tag{10}$$

Squaring both sides of (9) and neglecting terms of e 's having power greater than two we have

$$\begin{aligned}(l-S_y^2)^2 &\cong S_y^4[A^2\{1+e_0^2+\theta^2e_1^2+2e_0+2\theta e_1+4\theta e_0e_1-2\theta\theta_2e_1^2\}\\ &\quad +1-2A\{1+e_0+\theta(e_1+e_0e_1-\theta_2e_1^2)\}]\\ &\cong S_y^4[1+A^2\{1+2e_0+2\theta e_1+e_0^2+\theta(\theta-2\theta_2)e_1^2+4\theta e_0e_1\}\\ &\quad -2A\{1+e_0+\theta(e_1+e_0e_1-\theta_2e_1^2)\}].\end{aligned} \tag{11}$$

Taking expectation of both sides of (11), we get the *MSE* of l to the first degree of approximation [ignoring (fpc) term] as

$$\begin{aligned}MSE(l) &= S_y^4[1+A^2\{1+\gamma[(\lambda_{40}-1)+\theta(\lambda_{04}-1)(\theta-2\theta_2+4k)]\}\\ &\quad -2A\{1+\gamma\theta(\lambda_{04}-1)(k-\theta_2)\}].\end{aligned} \tag{12}$$

From (10) we have

$$\lim_{n\to\infty} B(l) = S_y^2(A-1). \tag{13}$$

It follows that the proposed class of estimators l is not consistent. To make it consistent we have to assume that *A*=1 or $((a+b)/(c+d))=1$. Thus under the condition *A*=1, we get the bias and *MSE* of the proposed class of consistent estimators l_C (say) to the first degree of approximation [ignoring (f.p.c.) term] respectively as

$$B(l_C) = \gamma S_y^2\theta(\lambda_{04}-1)(k-\theta_2), \tag{14}$$

$$MSE(l_C)=\gamma S_y^4\left[(\lambda_{40}-1)+\theta(\lambda_{04}-1)(\theta+2k)\right]. \tag{15}$$

The $MSE(l_C)$ at (15) is minimum when

$$\theta=-k=\theta_{opt}\text{ ,(say).} \tag{16}$$

Putting (16) in (15) we get the minimum *MSE* of l_C as

$$MSE_{\min}(l_C)=\gamma S_y^4[(\lambda_{40}-1)-(\lambda_{04}-1)k^2]$$
$$=\gamma S_y^4(\lambda_{40}-1)(1-\rho^{*2}), \tag{17}$$

where $\rho^*=\dfrac{(\lambda_{22}-1)}{\sqrt{(\lambda_{40}-1)(\lambda_{04}-1)}}\cong\dfrac{Cov(s_y^2,s_x^2)}{\sqrt{V(s_y^2)V(s_x^2)}}$.

Now, we consider the case when $A\neq 1$ or $(a+b)\neq(c+d)$.
Minimising (12) with respect to A, we get the optimum value of A as

$$A=\frac{\{1+\theta\gamma(\lambda_{04}-1)(k-\theta_2)\}}{\{1+\gamma[(\lambda_{40}-1)+\theta(\lambda_{04}-1)(\theta-2\theta_2+4k)]\}}=A_{opt}. \tag{18}$$

Substituting A_{opt} in (12) we get the minimum *MSE* of *l* (in inconsistent case) as

$$MSE_{\min}(l)=S_y^4\left[1-\frac{\{1+\gamma\theta(\lambda_{04}-1)(k-\theta_2)\}^2}{\{1+\gamma[(\lambda_{40}-1)+\theta(\lambda_{04}-1)(\theta-2\theta_2+4k)]\}}\right]. \tag{19}$$

We have also generated some new consistent and inconsistent members of the suggested class of estimators *l* which are summarized in Tables 2 and 3 respectively.

Constants				**Estimator**
a	***b***	***c***	***d***	
$\beta_2(x)$	ρ	$\beta_2(x)$	ρ	$l_{C1}=s_y^2\left(\dfrac{\beta_2(x)S_x^2+\rho s_x^2}{\beta_2(x)s_x^2+\rho S_x^2}\right)$
$\beta_2(x)$	C_x	$\beta_2(x)$	C_x	$l_{C2}=s_y^2\left(\dfrac{\beta_2(x)S_x^2+C_x s_x^2}{\beta_2(x)s_x^2+C_x S_x^2}\right)$
1	ρ^2	1	ρ^2	$l_{C3}=s_y^2\left(\dfrac{S_x^2+\rho^2 s_x^2}{s_x^2+\rho^2 S_x^2}\right)$
ρ	f	ρ	f	$l_{C4}=s_y^2\left(\dfrac{\rho S_x^2+f s_x^2}{\rho s_x^2+f S_x^2}\right)$
C_x	f	C_x	f	$l_{C5}=s_y^2\left(\dfrac{C_x S_x^2+f s_x^2}{C_x s_x^2+f S_x^2}\right)$

$\beta_2(x)$	1	$\beta_2(x)$	1	$l_{C6} = s_y^2\left(\dfrac{\beta_2(x)S_x^2 + s_x^2}{\beta_2(x)s_x^2 + S_x^2}\right)$

Table 2: Some new consistent members of the suggested class of estimators *l*.

Constants				Estimator
a	***b***	***c***	***d***	
Q_d	1	Q_1	1	$l_{IC1} = s_y^2\left(\dfrac{Q_d S_x^2 + s_x^2}{Q_1 s_x^2 + S_x^2}\right)$
Q_r	1	Q_a	1	$l_{IC2} = s_y^2\left(\dfrac{Q_r S_x^2 + s_x^2}{Q_a s_x^2 + S_x^2}\right)$
Q_a	Q_d	Q_3	1	$l_{IC3} = s_y^2\left(\dfrac{Q_a S_x^2 + Q_d s_x^2}{Q_3 s_x^2 + S_x^2}\right)$
$\beta_2(x)$	C_x	$\beta_2(x)$	ρ	$l_{IC4} = s_y^2\left(\dfrac{\beta_2(x)S_x^2 + C_x s_x^2}{\beta_2(x)s_x^2 + \rho S_x^2}\right)$
ρ	f	C_x	f	$l_{IC5} = s_y^2\left(\dfrac{\rho S_x^2 + f s_x^2}{C_x s_x^2 + f S_x^2}\right)$
$\beta_2(x)$	ρ	$\beta_2(x)$	1	$l_{IC6} = s_y^2\left(\dfrac{\beta_2(x)S_x^2 + \rho s_x^2}{\beta_2(x)s_x^2 + S_x^2}\right)$

Table 3: Some new members (which are not consistent) of the class of estimators *l*.

3. Efficiency comparisons

3.1 Case -I [When A =1or $(a+b)/(c+d)$=1]

When the optimum value θ_{opt} coincides with its exact value – *k*, then from (2), (5) and (17) we have

$$V(l_0\; or\; s_y^2) - MSE_{\min}(l_C) = \gamma S_y^4(\lambda_{40}-1)\rho^{*2} \geq 0\,, \tag{20}$$

and

$$\begin{aligned} MSE(l_1) - MSE_{\min}(l_C) &= \gamma S_y^4[(\lambda_{04}-1)(1-2k) + \rho^{*2}(\lambda_{40}-1)] \\ &= \gamma S_y^4[\sqrt{(\lambda_{04}-1)} - \rho^*\sqrt{(\lambda_{40}-1)}]^2 \geq 0\,. \end{aligned} \tag{21}$$

It follows from (20) and (21) that the proposed class of consistent estimators l_C is more efficient than the usual unbiased estimator $l_0 = s_y^2$ and Isaki's (1983) ratio type estimator l_1 at its optimum condition.

3.2 Case –II [When $A \neq 1$ or $(a+b) \neq (c+d)$]

When the optimum value θ_{opt} of θ does not coincide with its exact optimum value – *k*, then from (2) and (15) we have

$$MSE(l_C) < V(l_0) \text{ if}$$

$$\left.\begin{array}{ll} \text{either} & 0 < \theta < -2k \\ \text{or} & -2k < \theta < 0 \end{array}\right\} \tag{22}$$

or equivalently,

$$\min.\{0, -2k\} < \theta < \max.\{0, -2k\}. \tag{23}$$

Further, from (5) and (15) we have that

$$MSE(l_1) < MSE(l_C)$$

if

$$(1-2k) < \theta(\theta + 2k)$$

$$\left.\begin{array}{lll} \text{i.e. if} & \text{either} & (1-2k) < \theta < -1 \\ & \text{or} & -1 < \theta < (1-2k) \end{array}\right\} \tag{24}$$

or equivalently,

$$\min.\{(1-2k), -1\} < \theta < \max.\{-1, (1-2k)\} \tag{25}$$

Now, we consider the two different subclasses of the consistent estimators l_C :

$$l_{C_i} = s_y^2 \left(\frac{a_i S_x^2 + b_i s_x^2}{c_i s_x^2 + d_i S_x^2} \right) \tag{26}$$

and

$$l_{C_j} = s_y^2 \left(\frac{a_j S_x^2 + b_j s_x^2}{c_j s_x^2 + d_j S_x^2} \right), \tag{27}$$

where $\left(\frac{a_i + b_i}{c_i + d_i} \right) = 1$ and $\left(\frac{a_j + b_j}{c_j + d_j} \right) = 1$.

Then under the condition $\left(\frac{a_i + b_i}{c_i + d_i} \right) = 1$ and $\left(\frac{a_j + b_j}{c_j + d_j} \right) = 1$, from (15) the MSEs of l_{C_i} and l_{C_j} are respectively given by

$$MSE(l_{C_i}) = \gamma S_y^4 \left[(\lambda_{40} - 1) + \theta_i (\lambda_{04} - 1)(\theta_i + 2k) \right] \tag{28}$$

and

$$MSE(l_{C_j}) = \gamma S_y^4\left[(\lambda_{40}-1)+\theta_j(\lambda_{04}-1)(\theta_j+2k)\right] \quad (29)$$

where $\theta_i = (\theta_{1i}-\theta_{2i})$, $\theta_j = (\theta_{1j}-\theta_{2j})$, $\theta_{1i} = b_i/(a_i+b_i)$,
$\theta_{1j} = b_j/(a_j+b_j)$, $\theta_{2i} = c_i/(c_i+d_i)$ and $\theta_{2j} = c_j/(c_j+d_j)$.

From (28) and (29) we have

$$MSE(l_{C_i}) < MSE(l_{C_j})$$

if $\quad \theta_i(\theta_i+2k) < \theta_j(\theta_j+2k)$

i.e. if $\quad (\theta_i^2-\theta_j^2)+2k(\theta_i-\theta_j) < 0$

i.e. if

$$\text{either}\ (\theta_i+\theta_j+2k) < 0\,, \theta_i > \theta_j \quad (30)$$

$$\text{or}\quad (\theta_i+\theta_j+2k) > 0\,, \theta_i < \theta_j \quad (31)$$

4. Empirical study

In this section, we compare the proposed class of consistent/ inconsistent estimators l_C/l with other exiting estimators through a natural population data set [Singh and Chaudhary (1986, p. 108)] summarized in Table 4.

N	70	C_y	0.6254	Q_1	80.1500
n	25	S_x	140.8572	Q_2	160.3000
$\bar{Y}$	96.7000	C_x	0.8037	Q_3	225.0250
$\bar{X}$	175.2671	λ_{04}	7.0952	Q_r	144.8750
ρ	0.7293	λ_{40}	4.7596	Q_d	72.4375
S_y	60.7140	λ_{22}	4.6038	Q_a	152.5875

Table 4: The population data set.

We have computed the percent relative efficiencies (*PRE*s) of the proposed consistent estimators l_{Ci}, inconsistent estimators l_{ICi} (say), (i = 1, 2, …6) and the existing consistent estimators l_j, (j = 0, 1, 2, …, 11) (as given in Table 1) with respect to the usual unbiased estimator $l_0 = s_y^2$ using the following formula

$$PRE(\bullet,\, s_y^2) = \frac{V(s_y^2)}{MSE(\bullet)}\times 100, \quad (32)$$

where $(\bullet)$ stands for l_{Ci}, l_{ICi}, and l_j (i = 1, 2, …6; j = 0, 1, 2, …, 11) and finding are summarized in Table 5.

$PRE(l_0, s_y^2)$	100.00	$PRE(l_{C1}, s_y^2)$	194.78
$PRE(l_1, s_y^2)$	142.02	$PRE(l_{C2}, s_y^2)$	199.38
$PRE(l_2, s_y^2)$	142.12	$PRE(l_{C3}, s_y^2)$	176.82
$PRE(l_3, s_y^2)$	142.03	$PRE(l_{C4}, s_y^2)$	187.43
$PRE(l_4, s_y^2)$	142.02	$PRE(l_{C5}, s_y^2)$	199.03
$PRE(l_5, s_y^2)$	142.14	$PRE(l_{C6}, s_y^2)$	210.25
$PRE(l_6, s_y^2)$	143.10	$PRE(l_{IC1}, s_y^2)$	199.30
$PRE(l_7, s_y^2)$	145.04	$PRE(l_{IC2}, s_y^2)$	173.96
$PRE(l_8, s_y^2)$	143.97	$PRE(l_{IC3}, s_y^2)$	**229.05**
$PRE(l_9, s_y^2)$	143.00	$PRE(l_{IC4}, s_y^2)$	190.28
$PRE(l_{10}, s_y^2)$	144.07	$PRE(l_{IC5}, s_y^2)$	211.31
$PRE(l_{11}, s_y^2)$	146.16	$PRE(l_{IC6}, s_y^2)$	224.91

*bold numbers indicated the most efficient estimator.

Table 5: The *PRE*s of different estimators of S_y^2 with respect to s_y^2.

It is observed from Table 5 that the proposed consistent estimators l_{Ci} and inconsistent estimators l_{ICi}, (i = 1, 2, …6) proved to be better than the usual unbiased estimator s_y^2, the ratio estimator l_1 due to Isaki (1983), the estimators (l_2, l_3, l_4, l_5) due to Kadilar and Cingi (2006), the estimators ($l_6, l_7, l_8, l_9, l_{10}$) due to Subramani and Kumarapandiyan (2012) and the estimator l_{11} due to Khan and Shabbir (2013). Further, we note that the inconsistent estimator l_{IC3} is the best estimator in the sense of having largest PRE [= 229.05] among all the estimators discussed here.

5. Conclusion

We have suggested an improved class of ratio-type estimators of population variance using information on an auxiliary variable with their properties in simple random sampling. The suggested class of estimators encompasses many existing estimators of population variance such as the usual unbiased estimator and the estimators due to Isaki (1983), Kadilar and Cingi (2006), Subramani and Kumarapandiyan (2012) and Khan and Shabbir (2013). Some new consistent and inconsistent estimators of population variance have been also generated from the proposed class. It has been shown theoretically as well as empirically that the proposed class of estimators is more general and efficient than the existing estimators of the population variance. However, this conclusion should not be extrapolated due to limited empirical study.

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