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RATIO-CUM-PRODUCT ESTIMATOR OF POPULATION MEAN IN SYSTEMATIC SAMPLING USING KNOWN PARAMETERS OF AUXILIARY VARIATES

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Abstract

In this paper, a ratio-cum-product estimator of population mean in systematic sampling has been proposed using Kadilar and Cingi (2006) estimator. The bias and mean squared error of the proposed estimator has been obtained under large sample approximation. The proposed estimator has been compared with simple mean estimator, usual ratio and product estimators in systematic sampling given by Swain (1964) and Shukla (1971) respectively. An empirical study has been carried out to demonstrate the performance of the proposed estimator.

Key Words: Ratio Estimator, Product Estimator, Correlation Coefficient, Coefficient of Variation, Systematic Sampling, Bias, Mean Squired Error.

1. Introduction

In sample surveys, the auxiliary information is used at the estimation stage to improve the efficiency of the estimators of the population parameters. Out of many, ratio, product and regression estimators are good examples in this context. When the correlation between the study variate and auxiliary variate is positively high, the ratio method of estimation is used for estimating the population mean. On the other hand, if this correlation is negative, the product method of estimation envisaged by Robson (1957) is used.

Systematic sampling has got the nice feature of selecting the whole sample with just one random start. Apart from its simplicity, which is of considerable importance, this procedure in many situations provides estimator more efficient than simple random sampling and stratified tendon sampling for certain type of population from 1 to N in some order.

In systematic sampling, Swain (1964) defined ratio estimator while Shukla (1971) defined product estimator for the population mean. Kadilar and Cingi (2006) used coefficient variation of auxiliary variates and correlation coefficient between the study variate and auxiliary variate.

Singh and Tailor (2005) and Tailor and Sharma (2009) defined ratio-cum-product estimator for population mean using suitably chosen scalar α in simple random sampling that motivate authors to propose a ratio-cum-product type estimator of population mean in systematic sampling.

Let $U=\{U_1,U_2,...,U_N\}$ be the population of size N serially numbered from 1 to N . We assume that $N = nk$, where n and k are positively integers. Thus there will be k samples (clusters) each of size n . We select a sample at random out of k samples and observe the study variate y and auxiliary variate x for each and every unit selected the sample.

Let y_{ij} and x_{ij} $(i=1,2,...,k, j=1,2,...,n)$ denote the value of j^{th} unit in the i^{th} sample. The systematic sample means of study variate y and auxiliary variate x are respectively defined as

$$\bar{y}_{sys}=\frac{1}{n}\sum_{j=1}^{n}y_{ij} \tag{1.1}$$

and

$$\bar{x}_{sys}=\frac{1}{n}\sum_{j=1}^{n}x_{ij} . \tag{1.2}$$

It is to be noted that $\bar{y}_{sys}$ and $\bar{x}_{sys}$ are unbiased estimator of population mean $\bar{Y}$ and $\bar{X}$ respectively.

Swain (1964) studied the classical ratio estimator of population mean $\bar{Y}$ as

$$\bar{y}_{Rsy}=\bar{y}_{sys}\left(\frac{\bar{X}}{\bar{x}_{sys}}\right) . \tag{1.3}$$

Shukla (1971) defined classical product estimator of population mean in systematic sampling as

$$\bar{y}_{Psy}=\bar{y}_{sys}\left(\frac{\bar{x}_{sys}}{\bar{X}}\right) . \tag{1.4}$$

The biases and mean squared errors (MSEs) of $\bar{y}_{Rsy}$ and $\bar{y}_{Psy}$ to the first degree of approximation are respectively given by

$$B(\bar{y}_{Rsy})=\bar{Y}\left[\frac{1}{\bar{X}^2}V(\bar{x}_{sys})-\frac{1}{\bar{X}\bar{Y}}\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.5}$$

$$MSE(\bar{y}_{Rsy})=V(\bar{y}_{sys})+R^2V(\bar{x}_{sys})-2R\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}, \tag{1.6}$$

$$B(\bar{y}_{Psy})=\bar{Y}\left[\frac{1}{\bar{X}\bar{Y}}\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.7}$$

$$MSE(\bar{y}_{Psy})=V(\bar{y}_{sys})+R^2V(\bar{x}_{sys})+2R\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}. \tag{1.8}$$

where

$$V(\bar{y}_{sys}) = \left(\frac{N-1}{N}\right)\left(\frac{S^2{}_y}{n}\right)\left\{1+\rho_y(n-1)\right\} , \tag{1.9}$$

$$V(\bar{x}_{sys}) = \left(\frac{N-1}{N}\right)\left(\frac{S^2{}_x}{n}\right)\left\{1+\rho_x(n-1)\right\} , \tag{1.10}$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij}-\bar{X})^2 , \quad S_y^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{Y})^2$$

$$R = \frac{\bar{Y}}{\bar{X}}, \quad S_{yx} = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij}-\bar{X})(y_{ij}-\bar{Y}), \quad \rho_{yx} = \frac{S_{yx}}{S_y S_x} ,$$

$$\rho_x = 1-\frac{n}{(n-1)}\sigma_{w_x}^2\sigma_x^2 , \quad \rho_y = 1-\frac{n}{(n-1)}\sigma_{w_y}^2\sigma_y^2 ,$$

here $\sigma_x^2, \sigma_y^2, \sigma_z^2$ and $\sigma_{w_x}^2, \sigma_{w_y}^2, \sigma_{w_z}^2$ are defined as

$$\sigma_x^2 = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij}-\bar{X})^2 , \quad \sigma_y^2 = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{Y})^2 ,$$

$$\sigma_{w_x}^2 = \frac{1}{k}\sum_{i=1}^{k}\sigma_{wi}^2 = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2$$ and

$$\sigma_{w_y}^2 = \frac{1}{k}\sum_{i=1}^{k}\sigma_{wi}^2 = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{y}_i)^2 .$$

Sisodia and Dwivedi (1981) and Pandey and Dubey (1988) estimators can be defined in systematic sampling as

$$\hat{\bar{Y}}_{SDR}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}_{sys}+C_x}{\bar{x}_{sys}+C_x}\right) . \tag{1.11}$$

$$\hat{\bar{Y}}_{PDp}^{sys} = \bar{y}_{sys}\left(\frac{\bar{x}_{sys}+C_x}{\bar{X}+C_x}\right) . \tag{1.12}$$

The biases and mean squared errors (MSEs) of $\hat{\bar{Y}}_{SD}^{sys}$ and $\hat{\bar{Y}}_{PD}^{sys}$ are respectively, given by

$$B(\hat{\bar{Y}}_{SDR}^{sys}) = \bar{Y}\left[\lambda_1^2\frac{1}{\bar{X}^2}V(\bar{x}_{sys}) - \lambda_1\frac{1}{\bar{X}\,\bar{Y}}\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right] , \tag{1.13}$$

$$MSE(\hat{\bar{Y}}_{SDR}^{sys}) = V(\bar{y}_{sys}) + \lambda_1^2R^2V(\bar{x}_{sys}) - 2\lambda_1R\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})} , \tag{1.14}$$

$$B(\hat{\bar{Y}}_{PDP}^{sys}) = \bar{Y}\left[\lambda_1 \frac{1}{\bar{X}\,\bar{Y}} \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.15}$$

$$MSE(\hat{\bar{Y}}_{PDP}^{sys}) = V(\bar{y}_{sys}) + \lambda_1^2 R^2 V(\bar{x}_{sys}) + 2\lambda_1 R\rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}. \tag{1.16}$$

where $R = \frac{\bar{Y}}{\bar{X}}$ and $\lambda_1 = \frac{\bar{X}}{\bar{X} + C_x}$.

Singh and Tailor (2003) defined a ratio and product type estimators of population mean $\bar{Y}$ using correlation coefficient ρ_{yx} which can be defined as

$$\hat{\bar{Y}}_{STR}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X} + \rho_{yx}}{\bar{x}_{sys} + \rho_{yx}}\right) \tag{1.17}$$

and

$$\hat{\bar{Y}}_{STP}^{sys} = \bar{y}_{sys}\left(\frac{\bar{x}_{sys} + \rho_{yx}}{\bar{X} + \rho_{yx}}\right). \tag{1.18}$$

The biases and mean squared errors (MSEs) of proposed estimators $\hat{\bar{Y}}_{STR}^{sys}$ and $\hat{\bar{Y}}_{STP}^{sys}$ to the first degree of approximation are given by

$$B(\hat{\bar{Y}}_{STR}^{sys}) = \bar{Y}\left[\lambda_2^2 \frac{1}{\bar{X}^2} V(\bar{x}_{sys}) - \lambda_2 \frac{1}{\bar{X}\,\bar{Y}} \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.19}$$

$$MSE(\hat{\bar{Y}}_{STR}^{sys}) = V(\bar{y}_{sys}) + \lambda_2^2 R^2 V(\bar{x}_{sys}) - 2\lambda_2 R\rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}, \tag{1.20}$$

$$B(\hat{\bar{Y}}_{STP}^{sys}) = \bar{Y}\left[\lambda_2 \frac{1}{\bar{X}\,\bar{Y}} \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.21}$$

$$MSE(\hat{\bar{Y}}_{STP}^{sys}) = V(\bar{y}_{sys}) + \lambda_2^2 R^2 V(\bar{x}_{sys}) + 2\lambda_2 R\rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})} \tag{1.22}$$

where $R = \frac{\bar{Y}}{\bar{X}}$ and $\lambda_2 = \frac{\bar{X}}{\bar{X} + \rho_{yx}}$.

Kadilar and Cingi (2006) estimators of population mean in systematic sampling are defined as

$$\hat{\bar{Y}}_{KS1}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}C_x + \rho_{xy}}{\bar{x}_{sys}C_x + \rho_{xy}}\right) \tag{1.23}$$

and

$$.\hat{\bar{Y}}_{KS2}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}\rho_{xy} + C_x}{\bar{x}_{sys}\rho_{xy} + C_x}\right) \tag{1.24}$$

To obtain the first degree of approximation, the biases and mean squared errors (MSEs) of estimators $\hat{\bar{Y}}_{KS1}^{sys}$ and $\hat{\bar{Y}}_{KS2}^{sys}$ are

$$B(\hat{\bar{Y}}_{KS1}^{sys})_R = \bar{Y}\left[\lambda_3^2 \frac{1}{\bar{X}^2} V(\bar{x}_{sys}) - \lambda_3 \frac{1}{\bar{X}\,\bar{Y}} \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.25}$$

$$MSE(\hat{\bar{Y}}_{KS1}^{sys})_R = V(\bar{y}_{sys}) + \lambda_3^2 R^2 V(\bar{x}_{sys}) - 2\lambda_3 R \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}, \tag{1.26}$$

$$B(\hat{\bar{Y}}_{KS2}^{sys})_R = \bar{Y}\left[\lambda_4^2 \frac{1}{\bar{X}^2} V(\bar{x}_{sys}) - \lambda_4 \frac{1}{\bar{X}\,\bar{Y}} \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right], \tag{1.27}$$

$$MSE(\hat{\bar{Y}}_{KS2}^{sys})_R = V(\bar{y}_{sys}) + \lambda_4^2 R^2 V(\bar{x}_{sys}) + 2\lambda_4 R \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}. \tag{1.28}$$

where $R = \dfrac{\bar{Y}}{\bar{X}}$, $\lambda_3 = \dfrac{\bar{X}C_x}{\bar{X}C_x + \rho_{yx}}$ and $\lambda_4 = \dfrac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx} + C_x}$.

2. Proposed estimator

Motivated by Singh and Ruiz Espejo (2003), authors propose a ratio-cum-product estimator using Kadilar and Cingi (2006) estimators in systematic sampling as

$$\bar{y}_{TJ} = \bar{y}_{sys}\left[\alpha\left(\frac{\bar{X}\rho_{yx} + C_x}{\bar{x}_{sys}\rho_{yx} + C_x}\right) + (1-\alpha)\left(\frac{\bar{x}_{sys}\rho_{yx} + C_x}{\bar{X}\rho_{yx} + C_x}\right)\right] \tag{2.1}$$

where α is suitably chosen scalar.

To obtain bias and mean squared error of proposed estimator $\bar{y}_{TJ}$ we write,

$\bar{y}_{sys} = \bar{Y}(1+e_0)$, $\bar{x}_{sys} = \bar{X}(1+e_0)$ such that

$$E(e_0) = E(e_1) = 0$$

$$E(e_0^2) = \frac{1}{\bar{Y}^2} V(\bar{y}_{sys}),$$

$$E(e_1^2) = \frac{1}{\bar{X}^2} V(\bar{x}_{sys}),$$

and

$$E(e_0 e_1) = \frac{1}{\bar{Y}\,\bar{X}} \rho_{yx} \sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}.$$

Expressing (2.1) in terms of $e'_i s$ we have

$$\bar{y}_{TJ} = \bar{Y}(1+e_0)[\alpha(1+\lambda_4 e_1)^{-1} + (1-\alpha)(1+\lambda_4 e_1)]$$

Upto the first degree of approximation, the bias and mean squared error of the proposed estimator $\bar{y}_{TJ}$ are obtained as

$$B(\bar{y}_{TJ})=\bar{Y}\left[\left\{1+\lambda_4^2\frac{1}{\bar{X}^2}V(\bar{x}_{sys})-\lambda_4\frac{1}{\bar{X}\bar{Y}}\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right\}+(1-\alpha)\left\{1+\lambda_4\frac{1}{\bar{X}\bar{Y}}\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}\right\}-1\right] \quad (2.2)$$

and

$$MSE(\bar{y}_{TJ})=V(\bar{y}_{sys})+\lambda_4^2R^2V(\bar{x}_{sys})+4\lambda_4^2\alpha^2R^2V(\bar{x}_{sys})-4\lambda_4\alpha R\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}$$
$$-4\lambda_4^2\alpha R^2V(\bar{x}_{sys})+2\lambda_4R\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})} \quad (2.3)$$

where $R=\frac{\bar{Y}}{\bar{X}}$ and $\lambda_4=\frac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx}+C_x}$.

The value of α which minimizes mean squared error of the proposed estimator $\bar{y}_{TJ}$ is

$$\alpha=\frac{C}{B}$$

where

$$C=4\lambda_4^2R^2V(\bar{x}_{sys})$$

$$B=2\lambda_4^2R^2V(\bar{x}_{sys})+2\lambda_4R\rho_{yx}\sqrt{V(\bar{y}_{sys})V(\bar{x}_{sys})}$$

The minimum mean squared error of the proposed estimator $\bar{y}_{TJ}$ is

$$Min..MSE(\bar{y}_{TJ})=V(\bar{y}_{sys})(1-\rho_{yx}^2) \quad (2.4)$$

which is mean squared error of the regression estimator in systematic sampling.

3. Efficiency comparison of proposed estimator

Comparison of (1.6), (1.8), (1.9), (1.14), (1.16), (1.20), (1.22), (1.26), (1.28) and (2.3) shows that the proposed estimator $\bar{y}_{TJ}$ would be more efficient than

(i) $\bar{y}_{sys}$ if

$$A^2+B_1<0, \quad (3.1)$$

(ii) $\bar{y}_{Rsy}$ if

$$A^2+B_2<0, \quad (3.2)$$

(iii) $\bar{y}_{psy}$ if

$$A^2+B_3<0, \quad (3.3)$$

(iv) $(\hat{\bar{Y}}_{SD}^{sys})_R$ if

$$A^2+B_4<0, \quad (3.4)$$

(v) $(\hat{\bar{Y}}_{SD}^{sys})_P$ if

$$A^2+B_5<0, \quad (3.5)$$

(vi) $\hat{\bar{Y}}_{STR}^{sys}$ if

$$A^2 + B_6 < 0, \tag{3.6}$$

(vii) $\hat{\bar{Y}}_{STP}^{sys}$ if

$$A^2 + B_7 < 0, \tag{3.7}$$

(viii) $\hat{\bar{Y}}_{KS1R}^{sys}$ if

$$A^2 + B_8 < 0, \tag{3.8}$$

(ix) $MSE(\bar{y}_{TJ}) - MSE(\hat{\bar{Y}}_{KS2}^{sys})_R < 0$ if

$$A^2 + B_9 < 0, \tag{3.9}$$

where

$$A = \lambda_4 R - 2\lambda_4 \alpha R,$$

$$B_1 = 2\lambda_4 Rk - 4\lambda_4 \alpha Rk,$$

$$B_2 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - R^2 + 2Rk,$$

$$B_3 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - R^2 - 2Rk,$$

$$B_4 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - \lambda_1^2 R^2 + 2R\lambda_1 k,$$

$$B_5 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - \lambda_1^2 R^2 - 2R\lambda_1 k,$$

$$B_6 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - \lambda_2^2 R^2 + 2R\lambda_2 k,$$

$$B_7 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - \lambda_2^2 R^2 - 2R\lambda_2 k,$$

$$B_8 = 4\lambda_4 \alpha Rk + 2\lambda_4 Rk - \lambda_3^2 R^2 + 2R\lambda_3 k,$$

and

$$B_9 = 4\lambda_4 \alpha Rk - \lambda_4^2 R^2 + 4R\lambda_4 k.$$

Condition (3.1) to (3.9) are conditions under which proposed estimator $\bar{y}_{TJ}$ has less mean squared error than all other considered estimator respectively.

4. Optimum estimator with the $\alpha = \frac{C}{B}$

Optimum estimator with $\alpha = \frac{C}{B}$ is defined as

$$\hat{\bar{y}}_{TJ\alpha}^{(opt)} = \bar{y}_{sys}\left[\alpha_{(opt)}\left(\frac{\bar{X}\rho_{yx} + C_x}{\bar{x}_{sys}\rho_{yx} + C_x}\right) + (1-\alpha_{(opt)})\left(\frac{\bar{x}_{sys}\rho_{yx} + C_x}{\bar{X}\rho_{yx} + C_x}\right)\right] \tag{4.1}$$

With this is $\alpha_{(opt)}$, minimum mean squared error $\hat{\bar{y}}_{TJ\alpha}^{(opt)}$ is obtained as

$$Min..MSE(\hat{\bar{y}}_{TJ}^{(opt)}) = V(\bar{y}_{sys})(1-\rho_{yx}^2) \tag{4.2}$$

5. Efficiency comparisons for $\hat{\bar{y}}_{TJ\alpha}^{(opt)}$

Comparison of (1.6), (1.8), (1.9), (1.14), (1.16), (1.20), (1.22), (1.26), (1.28) and (2.4) shows that the proposed optimum estimator $\hat{\bar{y}}_{TJ\alpha}^{(opt)}$ would be more efficient than

(i) $\bar{y}_{sys}$

$$\rho_{yx}^2 > 0, \tag{5.1}$$

(ii) $\bar{y}_{Rsy}$

$$(k - R)^2 > 0, \tag{5.2}$$

(iii) $\bar{y}_{Psy}$

$$(k + R)^2 > 0, \tag{5.3}$$

(iv) $\hat{\bar{Y}}_{SDR}^{sys}$

$$(k - \lambda_1 R)^2 > 0, \tag{5.4}$$

(v) $\hat{\bar{Y}}_{SDP}^{sys}$

$$(k + \lambda_1 R)^2 > 0, \tag{5.5}$$

(vi) $\hat{\bar{Y}}_{STR}^{sys}$

$$(k - \lambda_2 R)^2 > 0, \tag{5.6}$$

(vii) $\hat{\bar{Y}}_{STP}^{sys}$

$$(k + \lambda_2 R)^2 > 0, \tag{5.7}$$

(viii) $\hat{\bar{Y}}_{KS1R}^{sys}$

$$(k - \lambda_3 R)^2 > 0, \tag{5.8}$$

(ix) $\hat{\bar{Y}}_{KS2R}^{sys}$

$$(k - \lambda_4 R)^2 > 0. \tag{5.9}$$

$\lambda_i = (i = 1,2,3,4)$ are already defined. Expressions (5.1) to (5.9) are conditions Which are always true. Thus the proposed estimator $\bar{y}_{TJ\alpha}^{(opt)}$ has less mean squared error than all other considered estimators.

6. Empirical study

To compare the proposed estimator $\bar{y}_{TJ}$ with other estimators empirically, we are considering two natural population data sets. Descriptions of populations are given below:

Population I [Source: Johanson and Wichard (2003), p. 275]

x:Male width, y:Male height,

$$\overline{Y} = 38.80\ ,\quad S_y^2 = 4.89\ ,\quad C_y = 0.06\ ,\ S_{xy} = 9.20\ ,$$
$$\overline{X} = 84.27\ ,\ S_x^2 = 23.79,\quad C_x = 0.06\ ,\ \rho_{yx} = 0.86\ ,$$
$$\rho_x = 0.77,\quad \rho_y = 0.59,\qquad N=15,\qquad n=3.$$

Population II [Source: Bhuyan (2005), p. 4]
x:Level of education of father(in completed Years)
y:Level of education of mother(in completed Years)

$$\overline{Y} = 10.93,\ S_y^2 = 26.50,\ C_y = 0.47,\quad S_{xy} = 16.72,$$
$$\overline{X} = 5.13,\ S_x^2 = 10.55,\ C_x = 0.63,\ \rho_{yx} = 0.88,$$
$$\rho_x = -0.09,\ \rho_y = 0.10,\ N=15,\qquad n=3.$$

Estimator	$\hat{\overline{Y}}_{sys}$	$MSE(\overline{y}_{Rsy})$	$MSE(\overline{y}_{TJ})$	$Min.MSE(\overline{y}_{TJ})$
Population I	100.00	303.85	337.45	337.45
Population II	100.00	372.20	459.70	459.70

Table 5.1: Percent Relative Efficiency of $\overline{y}_{sys}$, $\overline{y}_{Rsy}$ and $\overline{y}_{TJ}$ with respect to $\overline{y}_{sys}$

Section 4 and 5 provide the conditions under which proposed estimators have less mean squared error than mean squared error of other estimator. Table 5.1 shows that proposed estimator $\overline{y}_{TJ}$ has highest percent relative efficiency in comparison to other estimators. These proposed estimators are recommended for use in practice for the estimation of population mean.

References

1. Bhuyan, K.C. (2005). Multivariate Analysis and Its Application, Kolkata, India: New Central Book Agency (P) Ltd.
2. Johnson, R. A. and Wichern, D. W. (2003). Applied Multivariate Analysis, Prentic Hall India Private Limited, New Delhi.
3. Kadilar, C. and Cingi, H. (2006). Improvement in estimating the population mean in simple random sampling, Appl. Math. Lett., 19, p. 75-79.
4. Murty, M. N. (1964). Product method of estimation, Sankhya, A, 26, p. 294-307.
5. Pandey, B.N. and Dubey, V. (1988). Modified product estimator using coefficient of variation of auxiliary variable, Assam Statist. Rev., 2, p. 64-66.
6. Robson, D.S. (1957). Application of multivariate polykays to the theory of unbiased ratio-type estimation, J. Amer. Statist. Assoc., 52, p. 511 - 522.

7. Shukla, N. D. (1971). Systematic sampling and product method of estimation, Proceedings of all India Seminar on Demography and Statistics, B.H.U. Varanasi, India.
8. Singh, H. P. and Espejo, M. R. (2003). On linear regression and ratio product estimators of finite population mean, Statistician, 52(1), p. 59-67.
9. Singh, H. P. and Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean, Statistics in Transition, 6(4), p. 555-560.
10. Singh, H. P. and Tailor, R. (2005). Estimation of finite population mean with known coefficient of variation between auxiliary characters, Statistica, 65(3), p. 301-313.
11. Singh, H.P. and Jatwa, N.K. (2012). A class of exponential-type estimators in systematic sampling, Eco. Qulty. Ctrl, 27, p. 195-208.
12. Singh, H.P., Tailor, R. and Jatwa, N. K. (2011). Modified ratio and product estimators for population mean in systematic sampling, J. Modern. Appl. Statist. Meth., 10(2), p. 424-435.
13. Sisodia, B. V. S. and Dwivedi, V. K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable, Jour. Ind. Soc. Agri. Stat., 33(1), p. 13-18.
14. Swain, A.K.P.C. (1964). The use of systematic sampling ratio estimate, Jour. Ind. Stat. Assoc., 2, p. 160-164.
15. Tailor, R. and Sharma, B. K. (2009). A modified ratio-cum-product estimator of finite population mean using known coefficient of variation and coefficient of kurtosis, Statist. in Transi., 10(1), 15-24.