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ESTIMATION OF R=P(X>Y) FOR MULTICOMPONENT SYSTEMS USING EXPONENTIAL STRENGTH AND GAMMA STRESS DISTRIBUTIONS

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Abstract

In this paper, we present estimation of $R = P(X > Y)$, a measure of system reliability, for multicomponent systems with standby stress-strength model. The system survives if and only if the strength (X) is greater than the stress (Y), otherwise fails. The statistical model is formed for study of system reliability. Here, we assume that all the components are independent and follow the exponential strength and gamma stress distributions. The marginal and system reliability are computed for different stress-strength parameters. The variation in system reliability is shown graphically.

Key Words: Stress-Strength Model, Standby System, System Reliability, Exponential Distribution, Gamma Distribution.

1. Introduction

Let $f(x)$ and $g(y)$ denotes the probability density functions of independent continuous random variables X (strength) and Y (stress) respectively. The reliability R of the component is given by,

$$R = P(X > Y) = \int_{-\infty}^{\infty}\left(\int_{y}^{\infty} f(x)\,dx\right) g(y)\,dy = \int_{-\infty}^{\infty}\overline{F}(y)\,g(y)\,dy, \tag{1.1}$$

or

$$R = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{x} g(y)\,dy\right) f(x)\,dx = \int_{-\infty}^{\infty} G(x)\,f(x)\,dx, \tag{1.2}$$

Where $\overline{F}(y) = 1 - F(y)$.

The reliability is the measure of the system performance such that the system survives if strength exceeds stress; otherwise, system fails at any time. Hence, in stress-strength model, ‘Stress’ is the dominating factor. To increase the reliability of the system, standby system is to be preferable. In standby system, it is assumed that, each component faces same stress imposed on it, while taking the place of failed component. The system survives up to the last component works. If all the components fail then system fails.

Gogoi and Borah [1] have estimated reliability for exponential, gamma and lindley stress-strength distributions by considering stress-strength model for dissimilar distributions. Pandit and Srivastav [2] have studied the n-cascade reliability for exponential distribution. They considered exponential stress-strength distribution. Rekha and Shyam Sundar [3] have derived an expression of the reliability of the n-cascade system by considering attenuation factor with the same parameter value. For study, they considered exponential strength and gamma stress distributions. Sandhya and Umamaheshwari [4] have derived reliability for multi component systems when stress-strength follows exponential and mixture of two exponential distributions. Srinivasa Rao et al. [5] have estimated reliability for inverse Rayleigh distribution considering different scale parameters. Sriwastav and Kakati [6] used cascade system for reliability estimation by considering that the components stress-strength is similarly distributed. The purpose of this paper is to study the variations in system reliability for different parameter values in a multicomponent strength-stress based on X and Y being two independent random variables. In practice, strength and stress distributions are not similar, so we have considered exponential strength and gamma stress distributions.

In this paper, Section 2 contains Statistical model, in which the formula for marginal reliability is given. Reliability computations are discussed in Section 3. Here, two cases of exponential strength and gamma stress distributions are considered for computations of marginal and system reliabilities. Numerical and Graphical study along with various observations are given in Section 4. Overall conclusion is summarized in Section 5.

2. Statistical Model

Let $X_i; i = 1,2,...,n$ be the set of *n* independent random variable denotes the strength of the components arranged in order of activation respectively, having probability density function $f_i(x); i = 1,2,...,n.$ Also let $Y_i; i = 1,2,...,n$ be the another set of *n* independent random variable denotes the stress imposed on the *n* components respectively, having probability density function $g_i(y).$ The system survives up to failure of first (*n*-1) components i.e. $X_i < Y_i; i = 1,2,...,(n-1) \quad and \; X_n > Y_n$. Therefore, the system reliability, R_n is defined as,

$$R_n = \sum_{i=1}^{n} R(i), \tag{2.1}$$

Where, $R(i), i = 1,2,...,n$ is the marginal reliability. Therefore, the n^{th} marginal reliability is defined as,

$$R(n) = \int_{-\infty}^{\infty} F_1(y) g_1(y)\, dy \times \int_{-\infty}^{\infty} F_2(y) g_2(y)\, dy \times ... \times \int_{-\infty}^{\infty} F_{n-1}(y) g_{n-1}(y)\, dy \times \int_{-\infty}^{\infty} \overline{F_n}(y) g_n(y)\, dy \tag{2.2}$$

Where, $F_i(y) = \int_{-\infty}^{y} f_i(x)\, dx \quad ,i = 1,2,...,n. \quad and \quad \overline{F_i}(y) = 1 - F_i(y)\cdot$

3. Reliability Computations

In the following, we compute marginal and system reliability for Exponential strength and Gamma stress distributions by considering two cases;

(i) One - parameter Exponential strength and one-parameter Gamma stress,

(ii) Two - parameter Exponential strength and two-parameter Gamma stress.

Case (i): One-parameter Exponential strength and one-parameter Gamma stress

Let the random variable X (strength) follows Exponential distribution with parameter $'a_i'$. The probability density function is given by,

$$f_i(x;a) = a_i \exp(-a_i x), \quad x>0, \quad a_i>0, i=1,2,...,n, \tag{3.1}$$

Its distribution function is given by,

$$F_i(x) = 1-\exp(-a_i x)\ , \tag{3.2}$$

Let, another random variable Y (stress) follows Gamma distribution with parameter c_i. The probability density function is given by,

$$g_i(y;c) = \frac{1}{\Gamma c_i} y^{c_i-1}\exp(-y), \quad y>0, \quad c_i>0, i=1,2,...,n, \tag{3.3}$$

Then from equation (2.2), marginal reliabilities are calculated by,

$$\begin{aligned} R(1) &= P(X_1>Y_1) \\ &= \int_0^\infty \overline{F_1}(y)\, g_1(y)\, dy \\ &= \int_0^\infty e^{-a_1 y} \frac{1}{\Gamma c_1} y^{c_1-1} e^{-y} dy \\ &= \frac{1}{(1+a_1)^{c_1}} \end{aligned}$$

$$\begin{aligned} R(2) &= P(X_1<Y_1, X_2>Y_2) \\ &= \left[\int_0^\infty F_1(y) g_1(y)\, dy\right]\left[\int_0^\infty \overline{F_2}(y) g_2(y)\, dy\right] \\ &= \int_0^\infty (1-e^{-a_1 y})\frac{1}{\Gamma c_1} y^{c_1-1}e^{-y}dy \ \int_0^\infty e^{-a_2 y}\frac{1}{\Gamma c_2} y^{c_2-1}e^{-y}dy \\ &= \left[1-\frac{1}{(1+a_1)^{c_1}}\right]\left[\frac{1}{(1+a_2)^{c_2}}\right] \end{aligned}$$

Similarly,

$$R(3) = \left[1-\frac{1}{(1+a_1)^{c_1}}\right]\left[1-\frac{1}{(1+a_2)^{c_2}}\right]\left[\frac{1}{(1+a_3)^{c_3}}\right] \text{ and}$$

$$R(4) = \left[1-\frac{1}{(1+a_1)^{c_1}}\right]\left[1-\frac{1}{(1+a_2)^{c_2}}\right]\left[1-\frac{1}{(1+a_3)^{c_3}}\right]\left[\frac{1}{(1+a_4)^{c_4}}\right]$$

In general,

$$R(n) = \left[\frac{1}{(1+a_n)^{c_n}}\right]\prod_{i=1}^{n-1}\left[1-\frac{1}{(1+a_i)^{c_i}}\right], \tag{3.4}$$

Case (ii): Two-parameter Exponential strength and two-parameter Gamma stress

Let, random variable X (strength) follows two-parameter Exponential distribution with parameters a_i *and* b_i ; $i = 1,2,...,n$. The probability density function is given by,

$$f_i(x; a,b) = a_i \exp[-a_i(x-b_i)], \quad x > b_i, \quad a_i > 0, b_i \geq 0, i = 1,2,...,n., \tag{3.5}$$

Its distribution function is given by,

$$F_i(x) = 1 - \exp[-a_i(x-b_i)] \ , \tag{3.6}$$

Let, another random variable Y (stress) follows two-parameter Gamma distribution with parameters c_i *and* d_i ; $i = 1,2,...,n$. The probability density function is given by,

$$g_i(y; c,d) = \frac{d_i^{c_i}}{\Gamma c_i} y^{c_i - 1} \exp(-d_i y) , \quad y > 0, \quad c_i \geq 0, d_i > 0, i = 1,2,...,n., \tag{3.7}$$

Then from equation (2.2), marginal reliabilities are calculated as;

$$R(1) = P(X_1 > Y_1) = \int_0^\infty \overline{F_1}(y)\, g_1(y)\, dy$$

$$= \int_0^\infty e^{-a_1 y} \frac{d_1^{c_1}}{\Gamma c_1} y^{c_1 - 1} e^{-d_1 y} dy$$

$$= \left(\frac{d_1}{a_1 + d_1}\right)^{c_1}$$

$$R(2) = P(X_1 < Y_1, X_2 > Y_2) = \left[\int_0^\infty F_1(y)\, g_1(y)\, dy\right]\left[\int_0^\infty \overline{F_2}(y)\, g_2(y)\, dy\right]$$

$$= \int_0^\infty (1 - e^{-a_1 y}) \frac{d_1^{c_1}}{\Gamma c_1} y^{c_1 - 1} e^{-d_1 y} dy \int_0^\infty e^{-a_2 y} \frac{d_2^{c_2}}{\Gamma c_2} y^{c_2 - 1} e^{-d_2 y} dy$$

$$= \left[1 - \left(\frac{d_1}{a_1 + d_1}\right)^{c_1}\right]\left(\frac{d_2}{a_2 + d_2}\right)^{c_2}$$

Similarly,

$$R(3) = \left[1 - \left(\frac{d_1}{a_1 + d_1}\right)^{c_1}\right]\left[1 - \left(\frac{d_2}{a_2 + d_2}\right)^{c_2}\right]\left(\frac{d_3}{a_3 + d_3}\right)^{c_3} \text{ and}$$

$$R(4) = \left[1 - \left(\frac{d_1}{a_1 + d_1}\right)^{c_1}\right]\left[1 - \left(\frac{d_2}{a_2 + d_2}\right)^{c_2}\right]\left[1 - \left(\frac{d_3}{d_3 + d_3}\right)^{c_3}\right]\left(\frac{d_4}{a_4 + d_4}\right)^{c_4}$$

In general,

$$R(n) = \left[\left(\frac{d_n}{a_n + d_n}\right)^{c_n}\right]\prod_{i=1}^{n-1}\left[1 - \left(\frac{d_i}{a_i + d_i}\right)^{c_i}\right], \tag{3.8}$$

Here, we note that, the reliability function given in (3.8) is independent of location parameter b_i , $i = 1,2,...,n$ of two parameter exponential distribution.

4. Numerical and Graphical Study of Marginal and System Reliability

In the following, we perform numerical and graphical study by calculating marginal and system reliabilities for two cases. For both cases, (i) and (ii), we set the strength parameter as constant i.e. 1 and then progressive i. e. 1 to 5 where as stress parameter(s) progressive i.e. 0.1 to 0.5 and then constant i.e. 0.1. Then we calculate marginal reliabilities $R(i), i = 1,2,3,4$ using equations (3.4) and (3.8) respectively. After that, we compute system reliabilities $R_n; n = 1,2,3,4$ using equation (2.1) for both cases. We plot graphs by taking progressive values of strength and stress versus system reliability respectively. We note that, R (1) = R_1.

a_i	c_i	R(1)	R(2)	R(3)	R(4)	R_2	R_3	R_4
1	0.1	0.933033	0.062482	0.004184	0.00028	0.995515	0.999700	0.99998
1	0.2	0.870551	0.112692	0.014588	0.001888	0.983243	0.997831	0.999719
1	0.3	0.812252	0.152498	0.028631	0.005375	0.964751	0.993382	0.998757
1	0.4	0.757858	0.183509	0.044435	0.01076	0.941367	0.985803	0.996562
1	0.5	0.707107	0.207107	0.060660	0.017767	0.914214	0.974874	0.992641
2	0.2	0.802742	0.158348	0.031235	0.006161	0.961089	0.992324	0.998486
3	0.3	0.659754	0.224479	0.076378	0.025987	0.884233	0.960611	0.986598
4	0.4	0.525306	0.249360	0.118370	0.056189	0.774665	0.893035	0.949224
5	0.5	0.408248	0.241582	0.142956	0.084595	0.649830	0.792786	0.877381
2	0.1	0.895958	0.093217	0.009698	0.001009	0.989175	0.998874	0.999883
3	0.1	0.870551	0.112692	0.014588	0.001888	0.983243	0.997831	0.999719
4	0.1	0.851340	0.126560	0.018814	0.002797	0.977900	0.996715	0.999512
5	0.1	0.835959	0.137132	0.022495	0.003690	0.973090	0.995586	0.999276

Table 1: Marginal and System Reliability for One-parameter Exponential strength and One-parameter Gamma stress: (for strength parameter a_i and stress parameter c_i; $i = 1,2,3,4$)

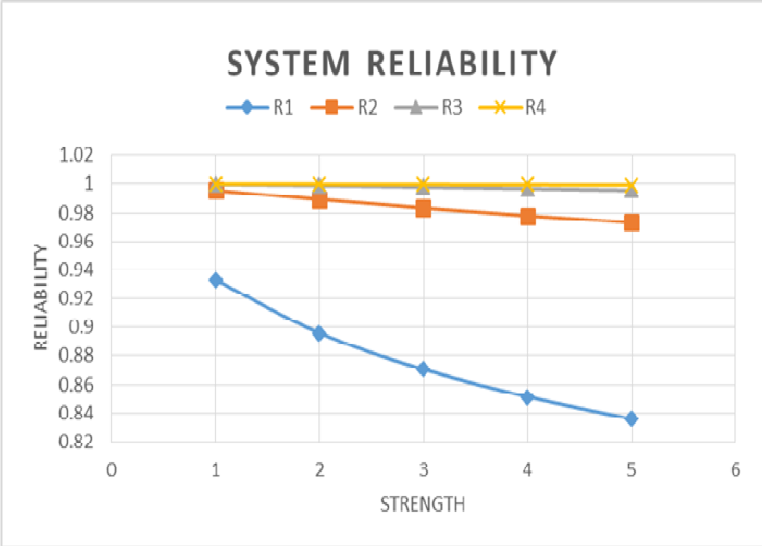

Fig. 1: System Reliability (for strength parameter a_i =1 to 5 and constant stress parameter c_i =0.1; $i = 1,2,3,4$)

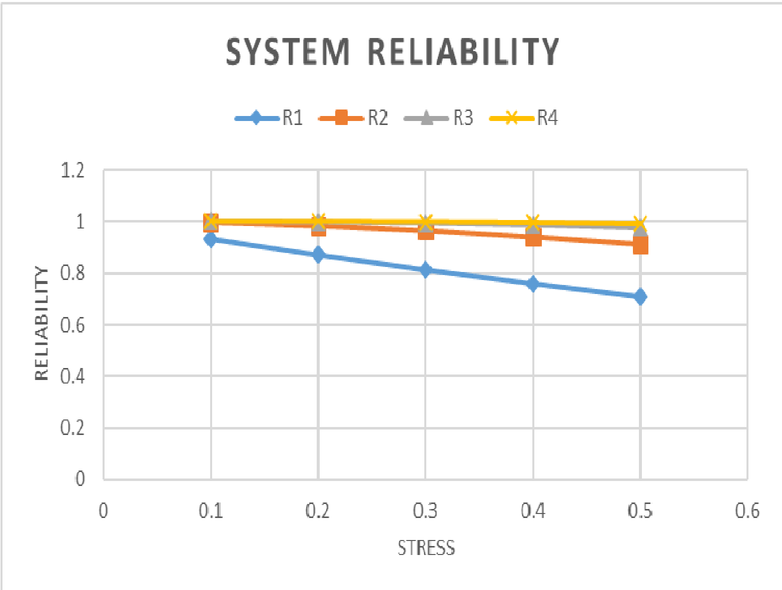

Fig. 2: System Reliability (for stress parameter c_i =0.1 to 0.5 and constant strength parameter a_i =1; $i = 1,2,3,4$)

a_i	c_i	d_i	R(1)	R(2)	R(3)	R(4)	R_2	R_3	R_4
1	0.1	0.1	0.786793	0.167750	0.035765	0.007625	0.954543	0.990308	0.997934
1	0.2	0.2	0.698827	0.210468	0.063387	0.019091	0.909295	0.972682	0.991773
1	0.3	0.3	0.644100	0.229235	0.081585	0.029036	0.873335	0.954920	0.983956
1	0.4	0.4	0.605861	0.238794	0.094118	0.037096	0.844654	0.938772	0.975868
1	0.5	0.5	0.577350	0.244017	0.103134	0.043589	0.821367	0.924501	0.968090
2	0.2	0.2	0.619044	0.235829	0.089840	0.034225	0.854872	0.944713	0.978938
3	0.3	0.3	0.487060	0.249833	0.128149	0.065733	0.736892	0.865041	0.930774
4	0.4	0.4	0.383215	0.236361	0.145784	0.089917	0.619577	0.765361	0.855278
5	0.5	0.5	0.301511	0.210602	0.147103	0.10275	0.512114	0.659217	0.761967
2	0.1	0.1	0.737527	0.193581	0.050810	0.013336	0.931108	0.981918	0.995254
3	0.1	0.1	0.709355	0.206170	0.059922	0.017416	0.915526	0.975448	0.992864
4	0.1	0.1	0.689797	0.213977	0.066376	0.02059	0.903774	0.970151	0.990741
5	0.1	0.1	0.674906	0.219408	0.071328	0.023188	0.894314	0.965642	0.988830

Table 2: Marginal and System Reliability for Two-parameter Exponential strength and Two-parameter Gamma stress: (for strength parameter a_i and stress parameters c_i , d_i ; $i = 1,2,3,4$)

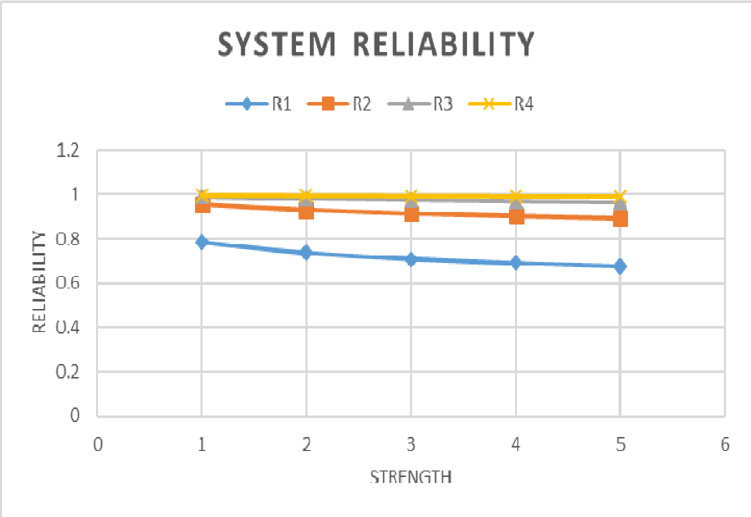

Fig. 3: System Reliability (for strength parameter a_i =1 to 5 and constant stress parameters c_i, d_i = 0.1; $i = 1,2,3,4$).

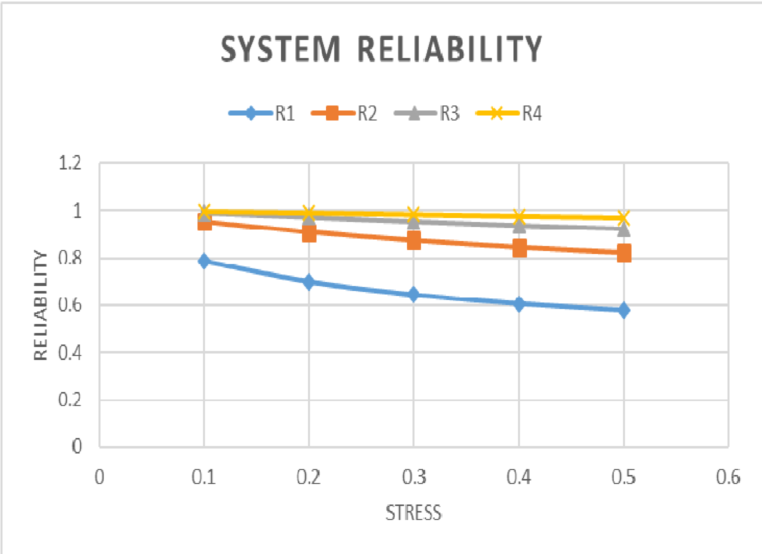

Fig. 4: System Reliability (for stress parameters c_i, d_i =0.1 to 0.5 and constant strength parameter a_i =1; $i = 1,2,3,4$).

From Table 1, 2 and Fig. 1-4, respectively, we observe that,

1. If strength parameter is constant and stress parameter(s) increases, the little change in system reliability R_4 occurs. In Table 1, reliability slightly decreases from 0.99998 to 0.992641 where, as in Table 2, reliability decreases from 0.997934 to 0.968090.
2. If strength and stress parameter(s) increases, the system reliability R_4 decreases. In Table 1, reliability decreases from 0.99998 to 0.877381, where as in Table 2, reliability decreases from 0.997934 to 0.761967.
3. If strength parameter increases and stress parameter(s) constants, the very little change in system reliability R_4 occurs. In Table 1, reliability very slightly decreases from 0.99998 to 0.999276, where as in Table 2, reliability decreases from 0.997934 to 0.988830.
4. In Fig. 1, the system reliability R_1 is far away from R_2, R_3, and R_4. As strength parameter increases, there is no significant variation in R_4. The system reliability R_4 stays very close to unity.
5. Fig. 2 shows that, R_1 is away from R_2, R_3, and R_4. The system reliability R_4 stays close to unity. As stress parameter increases, the system reliability R_4 slightly decreases.
6. Fig. 3 shows that, R_1 is away from R_2, R_3, and R_4. The system reliability R_4 stays very close to unity. As strength parameter increases, the system reliability R_4 very slowly decreases.
7. Fig. 4 shows that, the system reliabilities R_1, R_2, R_3, and R_4 are gradually decreases as stress parameters increases.
8. In all figures, we come across that the system reliability R_4 slightly decreases but stays near to unity.

5. Conclusions

(i) In this paper, we have given the reliability formula for multicomponent systems using exponential strength and gamma stress distributions by considering two cases.

(ii) If we want high system reliability R_4, then we have to set strength and stress parameter(s) systematically for both the cases. When we add standby components progressively we get system reliability very high, i.e. for case (i), if a_1=1and c_1=0.1, we get R_1=0.933033, R_2= 0.995515, R_3= 0.9997 and R_4= 0.99998. It means there is 6% increment in system reliability. For case (ii), if a_1=1 and c_1=d_1=0.1, we get R_1=0.786793, R_2= 0.954543, R_3= 0.990308 and R_4=0.997934. It means there is 21% increment in system reliability.

(iii) If we want system reliability R_4 stays close to unity then we have to keep stress parameter(s) constant and strength parameter progressively increasing for both cases.

(iv) We have come across that, for case (i), if a_1 varies from 1 to 5 and c_1 varies from 0.1 to 0.5, the system reliability R_4 decreases from 0.99998 to 0.877381 and for case (ii), if a_1 varies from 1 to 5 and c_1, d_1 varies from 0.1 to 0.5, the system reliability R_4 decreases from 0.997934 to 0.761967. It means there is 12% and 23% decrement respectively in the system reliability R_4 if strength and stress parameter(s) progressively increases.

(v) From the above discussion, we conclude that, in order to get high system reliability we have to choose strength parameter constant or progressively increasing and stress parameter(s) progressively increasing or constant respectively.

(vi) The systematic study of measure of system reliability, for multicomponent systems with standby stress-strength model, considering exponential strength and gamma stress distributions with different values of parameters helps to scientists and technocrats to decide exact strength and stress of component.

(vii) This study is useful to achieve high system reliability by setting strength and stress parameters properly for exponential strength and gamma stress distributions.

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