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GENERALIZED CHAIN RATIO REGRESSION AND CHAIN REGRESSION ESTIMATORS FOR THE POPULATION MEAN USING TWO AUXILIARY CHARACTERS IN THE PRESENCE OF NON-RESPONSE

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Abstract

Generalized chain ratio regression and chain regression estimators for the population mean using two auxiliary characters have been proposed in the presence of non-response. The expressions for the mean square error (MSE) of these estimators have been obtained for fixed sample sizes (n' and n) and also for fixed cost $C \le C_0$ .The optimum values of the first phase sample size n' and the sub sample of size n have been obtained for the fixed cost $C \le C_0$ and also for the specified precision $V = V_0'$.

Key Words: Population Mean, Study Variable, Auxiliary Variables, Mean Squared Error.

1. Introduction

In sample surveys related to human population, the information on the study variable is not available from all the units selected in the sample. In case of non-response in sample survey, Hansen and Hurwitz (1946) suggested a method of sub sampling from non-respondents for estimating the population mean. While estimating population parameters like the mean, total or ratio of two population mean in the sample surveys, the use of auxiliary information to improve the precision of the estimators is carried out. Further, the improvement in the efficiency of the estimator for population mean in the presence of non-response using known population mean of auxiliary character have been proposed by Rao (1986,90) , Khare and Srivastava (1996,1997).Further in the case of unknown population mean of the auxiliary character, Khare and Srivastava (1993,1995), Khare and Sinha (2002), Khare *et al*. (2008), Singh and Kumar (2010), Khare and Kumar (2009) and Khare and Srivastava (2010) have proposed different types of estimators for the estimation of population mean in the presence of non-response.

In the present paper, we have proposed generalized chain ratio regression and chain regression estimators for the population mean using two auxiliary characters in the presence of non-response. We obtained the expressions for mean square error of the proposed estimators in the case of fixed first phase sample (n'), second phase sample (n), sub sample fraction ($n_2/k, k>1$) and also in the case of fixed cost. The expressions for minimum expected total cost incurred in the proposed estimators are derived for the fixed variance. An empirical study has been given in the support of the present problem under investigation.

2. The Estimators

Let $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ denote the population mean of study character y, auxiliary character x and additional auxiliary character z having jth value Y_j, X_j and Z_j: $j = 1,2,3........N$. Supposed the population of size N is divided in N_1 responding units and N_2 not responding unit. According to Hansen and Hurwitz a sample of size n is taken from population of size N by using simple random sampling without replacement (SRSWOR) scheme of sampling and it has been observed that n_1 units respond and n_2 units do not respond. Again by making extra effort, a sub sample of size $r(= n_2 k^{-1})$ is drawn from n_2 non-responding unit and collect information on r units for study character y. Hence the estimator for $\bar{Y}$ based on $n_1 + r$ units on study character y is given by:

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}'_2 \tag{2.1}$$

Where n_1 and n_2 are the responding and non-responding units in a sample of size n selected from population of size N by SRSWOR method of sampling. $\bar{y}_1$ and $\bar{y}'_2$ are the means based on n_1 and r units selected from n_2 non-responding units by SRSWOR methods of sampling.

Similarly we can also define estimator for population mean $(\bar{X})$ of auxiliary character x based on n_1 and r unit respectively, which is given as;

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}'_2 \tag{2.2}$$

Variance of the estimators $\bar{y}^*$ and $\bar{x}^*$ are given by

$$V(\bar{y}^*) = \frac{f}{n}S_y^2 + \frac{W_2(k-1)}{n}S_{y(2)}^2 \tag{2.3}$$

And

$$V(\bar{x}^*) = \frac{f}{n}S_x^2 + \frac{W_2(k-1)}{n}S_{x(2)}^2 \tag{2.4}$$

where $f = 1 - \frac{n}{N}, W_2 = \frac{N_2}{N}$, $(S_y^2, S_{y(2)}^2)$ and $(S_x^2, S_{x(2)}^2)$ are population mean squares of y and x for entire population and non-responding part of population.

In case when the population mean of the auxiliary character is unknown, we select a larger first phase sample of size n' units from a population of size N units by using simple random sample without replacement (SRSWOR) method of sampling. Further second phase sample of size n (i.e. $n < n'$) is drawn from n' units by using SRSWOR method of sampling and variable y under investigation is measured n_1 responding and n_2 non-responding units. Again a sub sample of size $r\,(n_2 / k, k > 1)$ is drawn from n_2 non-responding units and collect information on r units by personal interview. Hence the estimator for $\overline{Y}$ based on $n_1 + r$ units is give by equation (2.1).
In this case two phase sampling ratio, product and regression estimators for population mean $\overline{Y}$ using one auxiliary character in the presence of non-response have been proposed by Khare and Srivastava (1993,1995) which are given as follows:

$$T_1 = \overline{y}^* \frac{\overline{x}'}{\overline{x}^*}, \qquad T_2 = \overline{y}^* \frac{\overline{x}'}{\overline{x}} \tag{2.5}$$

$$T_3 = \overline{y}^* + b^*\left(\overline{x}' - \overline{x}^*\right), \qquad T_4 = \overline{y}^* + b^{**}\left(\overline{x}' - \overline{x}\right) \tag{2.6}$$

where $\overline{x}^* = \frac{n_1}{n}\overline{x}_1 + \frac{n_2}{n}\overline{x}_2'$, $\overline{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$, $\overline{x}' = \frac{1}{n'}\sum_{j=1}^{n'} x_j$, $b^* = \frac{\hat{S}_{yx}}{\hat{S}_x^2}$, $b^{**} = \frac{\hat{S}_{yx}}{s_x^2}$,

$$s_x^2 = \sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$$

$\hat{S}_{yx}$ and $\hat{S}_x^2$ are estimates of S_{yx} and S_x^2 based on $n_1 + r$ units.
Further Singh and Kumar (2010) proposed difference type estimator using auxiliary character in the presence of non-response which is given as follows:

$$T_5 = \overline{y}^* + b_1\left(\overline{x} - \overline{x}^*\right) + b_2\left(\overline{x}' - \overline{x}\right) \tag{2.7}$$

where b_1 and b_2 are constants.

In case when $\overline{X}$ is not known than we may also use an additional auxiliary character z with known population mean $\overline{Z}$. In this case, it is assumed that the variable z is also corrected to y than x i.e, ($\rho_{yz} < \rho_{yx}$), x and z are variables such that z is more cheaper than x. In this case some estimators for population of the study character have been proposed by Chand (1975), Kiregyera (1980,84), Srivasatava *et al*. (1990), Khare *et al.* (2010) and Khare & Kumar (2011). In case of non-response on the study character, the chain regression type and generalized chain type estimators for the population mean in the presence of non-response have been proposed by Khare & Kumar (2010) and Khare *et al*. (2011). In this case we have proposed generalized chain regression and chain ratio regression estimators for the population mean using two auxiliary characters in the presence of non-response which are given as follows:

$$T_6 = \overline{y}^* + b_1\left(\overline{x} - \overline{x}^*\right) + b_2\left(\overline{x}' - \overline{x}\right) + b_3\left(\overline{Z} - \overline{z}'\right) \tag{2.8}$$

and

$$T_7 = \left[\bar{y}^* + a_1\left(\bar{x} - \bar{x}^*\right) + a_2\left(\bar{x}' - \bar{x}\right)\right]\left[\frac{\bar{z}'}{\bar{Z}}\right]^{\alpha} \tag{2.9}$$

where $b_1, b_2, b_3,\ a_1,\ a_2$ and α are constants, $\bar{Z}$ and $\bar{z}'$ are the means based on population of size N units and first phase sample of size n' units selected from population of size N by SRSWOR method.

3. Mean Square Errors of the Estimators T_6 and T_7

Using the large sample approximations, the expressions for the mean square errors of the estimators T_6 and T_7 up to the terms of order (n^{-1}) are given by-

$$MSE(T_6) = \left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\bar{Y}^2C_y^2 + b_2^2\bar{X}^2C_x^2 - 2\bar{X}\bar{Y}b_2C_{yx}\right\} + \left(\frac{1}{n'} - \frac{1}{N}\right)\left\{\bar{Y}^2C_y^2 + b_3^2\bar{Z}^2C_z^2 - 2\bar{Y}\bar{Z}b_3C_{yz}\right\}$$
$$+ \frac{W_2\,(K-1)}{n}\left\{\bar{Y}^2C_{y(2)}^2 + b_1^2\bar{X}^2C_{x(2)}^2 - 2\bar{X}\bar{Y}b_1C_{yx(2)}\right\} \tag{3.1}$$

$$MSE(T_7) = \left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\bar{Y}^2C_y^2 + a_2^2\bar{X}^2C_x^2 - 2\bar{X}\bar{Y}a_2C_{yx}\right\} + \left(\frac{1}{n'} - \frac{1}{N}\right)\left\{\bar{Y}^2C_y^2 + \alpha^2\bar{Y}^2C_z^2 + 2\alpha\bar{Y}^2C_{yz}\right\}$$

$$+ \frac{W_2\,(K-1)}{n}\left\{\bar{Y}^2C_{y(2)}^2 + a_1^2\bar{X}^2C_{x(2)}^2 - 2\bar{X}\bar{Y}a_1C_{yx(2)}\right\} \tag{3.2}$$

The optimum values of $(b_1, b_2 \text{ and } b_3)$ which minimized the T_6 and the values of $(a_1,\ a_2 \text{ and } \alpha)$ which minimized the T_7 are given as follows:

$$b_{1opt} = \frac{\bar{Y}\rho_{yx(2)}}{\bar{X}}\frac{C_{y(2)}}{C_{x(2)}} = a_{1opt} = Q_1, \qquad b_{2opt} = \frac{\bar{Y}\rho_{yx}}{\bar{X}}\frac{C_y}{C_x} = a_{2opt} = Q_2, \tag{3.3}$$

$$b_{3opt} = \frac{\bar{Y}\rho_{yz}}{\bar{Z}}\frac{C_y}{C_z} = Q_3 \qquad \text{and} \qquad \alpha_{opt} = -\frac{\rho_{yz}C_y}{C_z} = Q_4 \tag{3.4}$$

So, the minimum mean square error of the proposed estimators T_6 and T_7 for the population values of (b_1, b_2, b_3) and $(a_1,\ a_2, \alpha)$ are given as follows:

$$: MSE(T_6)\min = V(\bar{y}^*) - \bar{Y}^2\left\{\left(\frac{1}{n'} - \frac{1}{N}\right)\rho_{yz}^2C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2C_y^2 + \frac{W_2(k-1)}{n}\rho_{yx(2)}^2C_{y(2)}^2\right\} \tag{3.5}$$

and

$$MSE(T_7)\min = V(\bar{y}^*) - \bar{Y}^2\left\{\left(\frac{1}{n'} - \frac{1}{N}\right)\rho_{yz}^2C_y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2C_y^2 + \frac{W_2(k-1)}{n}\rho_{yx(2)}^2C_{y(2)}^2\right\} \tag{3.6}$$

Mean square errors of the estimators $T_1, T_2, T_3,\ T_4$ and T_5 are given as follows:

$$MSE(T_1) = V(\bar{y}^*) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left(C_x^2 - 2\rho_{yx}C_yC_x\right) + \frac{W_2(k-1)}{n}\left(C_{x(2)}^2 - 2\rho_{yx(2)}C_{y(2)}C_{x(2)}\right)\right]\bar{Y}^2 \tag{3.7}$$

$$MSE(T_2) = V(\bar{y}^*) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left(C_x^2 - 2\rho_{yx}C_yC_x\right)\right]\bar{Y}^2 \tag{3.8}$$

$$MSE(T_3) = V(\bar{y}^*) - \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2 C_y^2 + \frac{W_2(k-1)}{n}\left\{B^2 C_{x(2)}^2 - 2B\, C_{yx(2)}\right\}\right] \tag{3.9}$$

$$MSE(T_4) = V(\bar{y}^*) - \bar{Y}^2\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2 C_y^2 \tag{3.10}$$

and

$$MSE(T_5) = V(\bar{y}^*) - \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2 C_y^2 + \frac{W_2(k-1)}{n}\rho_{yx(2)}^2 C_{y(2)}^2\right] \tag{3.11}$$

where $V(\bar{y}^*) = \bar{Y}^2\left\{\frac{f}{n}C_y^2 + \frac{W_2(k-1)}{n}C_{y(2)}^2\right\}$ and $B = \frac{\bar{Y}\rho_{yx}C_y}{\bar{X}C_x}$

4. Comparisons of the Proposed Estimators T_6 and T_7 with Relevant Estimators

$$MSE(T_6) = MSE(T_7) < V(\bar{y}^*) \text{ if} \tag{4.1}$$

$0 < b_1 < 2Q_1$ for $Q_1 > 0$; $0 < b_2 < 2Q_2$ for $Q_2 > 0$;
$0 < b_3 < 2Q_3$ for $Q_3 > 0$;

$$MSE(T_6) = MSE(T_7) < MSE(T_5) \text{ if} \tag{4.2}$$

$0 < b_3 < 2Q_3$ for $Q_3 > 0$;

$$MSE(T_6) = MSE(T_7) < V(T_2) \text{ if} \tag{4.3}$$

$0 < b_1 < 2Q_1$ for $Q_1 > 0$; $0 < b_2 < 2Q_2$ for $Q_2 > 0$;
$0 < b_3 < 2Q_3$ for $Q_3 > 0$;

$$MSE(T_6) = MSE(T_7) < V(T_1) \text{ if} \tag{4.4}$$

$0 < b_1 < 2Q_1$ for $Q_1 > 0$; $0 < b_2 < 2Q_2$ for $Q_2 > 0$;
$0 < b_3 < 2Q_3$ for $Q_3 > 0$;

5. Determination of n', n and k for the Fixed Cost $C \le C_0$

Let us assume that C_0 be the total cost (fixed) of the survey apart from overhead cost. The expected total cost of the survey apart from overhead cost is written as follows:

$$C = (c_1' + c_2')n' + n\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k}\right) \tag{5.1}$$

where

c_1' - the cost per unit of obtaining information on auxiliary character x at the first phase.

c_2' - the cost per unit of obtaining information on additional auxiliary character z at the first phase.

c_1 - the cost per unit of mailing questionnaire/visiting the unit at the second phase.

c_2 - the cost per unit of collecting, processing data obtained from n_1 responding units.

c_3 - the cost per unit of obtaining and processing data (after extra efforts) for the sub sampling units.

and $W_1 = N_1 / N$, $W_2 = N_2 / N$

The expression for, $MSE(T_6)$ and $MSE(T_6)$ can be express in terms of V_{0j}, V_{1j}, V_{2j} and V_{3j} which is given as;

$$MSE(T_j)_{\min} = \frac{V_{0j}}{n} + \frac{V_{1j}}{n'} + \frac{k\,V_{2\mathrm{j}}}{n} - \frac{V_{3j}}{N} \quad \text{, j=6,7} \tag{5.2}$$

where V_{0j}, V_{1j}, V_{2j} and V_{3j} are the coefficients $\frac{1}{n}$, $\frac{1}{n'}$, $\frac{k}{n}$ and $\frac{1}{N}$ in the expression of $MSE(T_j)$.

To find the optimum values of n', n, k and minimizing $MSE(T_j)$ for the fixed cost $C \le C_0$ a function ϕ is defined.

$$\phi = MSE(T_j)_{\min} + \lambda_j \left(C - C_0\right), \quad \text{j=6, 7} \qquad 5.3)$$

where λ_j is the Lagrange's multiplier.

The optimum values of n', n and k are obtained after differentiating ϕ with respect to n', n, k and equating zero, which are given as

$$n'_{opt} = \sqrt{\frac{V_{1j}}{\lambda_j (c_1' + c_2')}} \tag{5.4}$$

$$n_{opt} = \sqrt{\frac{(V_{0j} + k_{opt}\ V_{2j})}{\lambda_j \left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}}\right)}} \tag{5.5}$$

and

$$k_{opt} = \sqrt{\frac{V_{0j} c_3 W_2}{V_{2j}(c_1 + c_2 W_1)}} \tag{5.6}$$

where

$$\sqrt{\lambda_j} = \frac{1}{C_0}\left[\sqrt{V_{1j}(c_1' + c_2')} + \sqrt{(V_{0j} + k_{opt}V_{2j})\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}}\right)}\right] \tag{5.7}$$

The minimum value of $MSE(T_j)$ are obtained as follows:

$$MSE(T_j)_{\min} = \frac{1}{C_0}\left[\sqrt{V_{1j}(c_1' + c_2')} + \sqrt{(V_{0j} + k_{opt}V_{2j})\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}}\right)}\right]^2 - \frac{V_{3j}}{N} \tag{5.8}$$

Now neglecting the term of O (N^{-1}), we have

$$MSE(T_j)_{\min} = \frac{1}{C_0}\left[\sqrt{V_{1j}(c_1' + c_2')} + \sqrt{(V_{0j} + k_{opt}V_{2j})\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k_{opt}}\right)}\right]^2 \tag{5.9}$$

6. Determination of n', n and k for the Specified Precision $V = V_0'$

Let V_0' be the variance of the estimator T_j which is fixed in advance, so we have

$$V_0' = \frac{V_{0j}}{n} + \frac{V_{1j}}{n'} + \frac{kV_{2j}}{n} - \frac{V_{3j}}{N} \tag{6.1}$$

To find the optimum values of n', n, k and minimum expected total cost, we define a function ψ which is give as-

$$\psi = (c_1' + c_2')n' + n\left(c_1 + c_2 W_1 + c_3 \frac{W_2}{k}\right) + \mu_j (MSE(T_j)_{\min} - V_0') \tag{6.2}$$

where μ_j is the Lagrange's multiplier.

After differentiating ψ with respect to n', n, k and equating to zero, we find the optimum value of n', n and k which are given as;

$$n' = \sqrt{\frac{\mu_j V_{1j}}{(c_1' + c_2')}} \tag{6.3}$$

$$n = \sqrt{\frac{\mu_j (V_{0j} + k_{opt} V_{2j})}{\left(c_1 + c_2 W_1 + c_3 \dfrac{W_2}{k_{opt}} \right)}} \tag{6.4}$$

and

$$k_{opt} = \sqrt{\frac{V_{0j} W_2 c_3}{V_{2j} (c_1 + c_2 W_1)}} \tag{6.5}$$

Where

$$\sqrt{\mu_j} = \frac{\left[\sqrt{V_{1j} (c_1' + c_2')} + \sqrt{(V_{0j} + k_{opt} V_{2j}) \left(c_1 + c_2 W_1 + c_3 \dfrac{W_2}{k_{opt}} \right)} \right]}{V_0' + \dfrac{V_{3j}}{N}} \tag{6.6}$$

The minimum expected total cost to be incurred on the use of T_j for the specified variance V_0' will be given by-

$$C_{j\min} = \frac{\left[\sqrt{V_{1j} (c_1' + c_2')} + \sqrt{(V_{0j} + k_{opt} V_{2j}) \left(c_1 + c_2 W_1 + c_3 \dfrac{W_2}{k_{opt}} \right)} \right]^2}{V_0' + \dfrac{V_{3j}}{N}} \quad j = 6,7 \tag{6.7}$$

Now neglecting the terms of O (N^{-1}), we have

$$C_{j\min} = \frac{\left[\sqrt{V_{1j} (c_1' + c_2')} + \sqrt{(V_{0j} + k_{opt} V_{2j}) \left(c_1 + c_2 W_1 + c_3 \dfrac{W_2}{k_{opt}} \right)} \right]^2}{V_0'} \tag{6.8}$$

7. An Empirical Study

To illustrate the results we consider the data earlier consider by Khare and Sinha (2007). The description of the population is given below:

Data Set- The data on physical growth of upper socio-economic group of 95 school children of Varanasi under an ICMR study, Department of Pediatrics, B.H.U., during 1983-84 has been taken under study. The first 25% (i. e. 24 children) units have been considered as non-responding units. Here we have taken the study character (y), auxiliary character (x) and the additional auxiliary character (z) are taken as follows:

y: weight (in kg.) of the children

x: skull circumference (in cm) of the children

z : chest circumference (in cm) of the children.

The values of the parameters of the y, x and z characters for the given data are given as follows:

$\bar{Y} = 19.4968$, $\bar{Z} = 51.1726$, $\bar{X} = 55.8611$, $C_y = 0.15613$, $C_z = 0.03006$, $C_x = 0.05860$, $C_{y(2)} = 0.12075$, $C_{z(2)} = 0.02478$, $C_{x(2)} = 0.05402$, $\rho_{yz} = 0.328$, $\rho_{yx} = 0.846$, $\rho_{xz} = 0.297$, $\rho_{xz(2)} = 0.570$, $W_2 = 0.25$, $W_1 = 0.74$, $N = 95, n = 35$

Estimators	$1/k$		
	1/4	1/3	1/2
$\bar{y}^*$	100 (0.28597)*	100 (0.24638)	100 (0.20679)
T_1	171(0.16686)	169 (0.14517)	167 (0.12348)
T_2	130 (0.22055)	136 (0.18096)	146 (0.14138)
T_3	197 (0.14500)	204 (0.12082)	214 (0.09664)
T_4	150 (0.19123)	162 (0.15164)	185 (0.11205)
T_5	211(0.13558)	215 (0.11454)	221 (0.09350)
$T_6 = T_7$	230 (0.12437)	233 (0.10582)	237 (0.08726)

*Figures in parenthesis give the MSE (.).

Table 7.1 Relative efficiency (in %) of the estimators with respect to $\bar{y}^*$ for the fixed values of n', n and different values of k (N =95, n' =70 and n =35)

From table 7.1, we obtained that for fixed sample sizes (n', n), the proposed estimators T_6 and T_7 are more efficient in comparison to the efficiency of the estimators $\bar{y}^*$, T_1, T_2, T_3, T_4 and T_5. The estimator T_6 is equally efficient to the estimator T_7 .The MSE of all the estimators decrease as the value of k decrease. It is observed that the relative efficiency of T_2 to T_7 increasing w.r.to $\bar{y}^*$ for decreasing value of k. But the relative efficiency of T_1 decrease as k decrease. This is due to reason that the MSE of T_2 to T_7 decreasing with the faster rate in comparison to T_1 as k decreases.

Estimators	k_{opt}	n'_{opt}	n_{opt}	Efficiency
$\bar{y}^*$	2.67	---	30	100 (0.38429)*
T_1	2.55	64	23	119 (0.32419)
T_2	1.73	61	20	108(0.35541)
T_3	1.63	87	17	150 (0.25623)
T_4	1.07	85	14	144 (0.26682)
T_5	1.96	89	19	155 (0.24772)
$T_6 = T_7$	1.96	77	19	157 (0.24493)

*Figures in parenthesis give the MSE (.).

Table 7.2 Relative efficiency (in %) of the estimators with respect to $\bar{y}^*$ (for the fixed cost $C \le C_0$ =*Rs.220*, c'_1=*Rs. 0.75*, c'_2=*Rs. 0.10*, c_1=*Rs. 2*, c_2=*Rs. 4*, c_3=*Rs. 25*).

From table 7.2, we obtained that for the fixed cost $C \le C_0$ the proposed estimators T_6 and T_7 are more efficient in comparison to the estimators $\bar{y}^*$, T_1, T_2, T_3, T_4 and T_5.The estimators T_6 and T_7 have equally efficiencies.

Estimators	k_{opt}	n'_{opt}	n_{opt}	Expected Cost (in Rs.)
$\bar{y}^*$	2.68	---	45	331
T_1	2.55	82	29	279
T_2	1.73	85	28	306
T_3	1.63	87	18	221
T_4	1.07	89	15	229
T_5	1.96	86	18	213
$T_6 = T_7$	1.96	74	18	210

Table 7.3 Expected cost of the estimators for the specified variance $V'_0 = .25564$: (c'_1=*Rs. 0.75*, c'_2=*Rs. 0.10*, c_1=*Rs. 2*, c_2=*Rs. 4*, c_3=*Rs. 25*)

From table 7.3, we obtained that for the specified variance the proposed estimators T_6 and T_7 have less cost in comparison to the cost incurred in the estimators $\bar{y}^*$, T_1, T_2, T_3, T_4 and T_5.The estimators T_6 and T_7 have equal cost.

8. Conclusion

(i) The information on additional auxiliary character and optimum values of (b_1, b_2, b_3, α) increase the efficiency of the proposed estimators in comparison to the corresponding estimators in case of the fixed sample sizes (n', n) and also in case of the fixed cost.

(ii) The information on additional auxiliary character and optimum values of (b_1, b_2, b_3, α) also reduce the cost of proposed estimators in comparison to cost incurred in corresponding estimators for the specified precision.

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