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SYSTEMATIC SAMPLING FOR NON-LINEAR TREND IN MILK YIELD DATA

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Abstract

The present paper utilizes systematic sampling procedures for milk yield data exhibiting some non-linear trends. The best fitted mathematical forms of non-linear trend present in the milk yield data are obtained and the expressions of average variances of the estimators of population mean under simple random, usual systematic and modified systematic sampling procedures have been derived for populations showing non-linear trend. A comparative study is made among the three sampling procedures for five data sets by calculating average variances using best fitted trend equations. Usual systematic sampling is found more precise than simple random and modified systematic sampling for four data sets whereas modified systematic sampling is found better than the other two procedures for one data set. Estimates of parameters for these trend functions are obtained with the help of SAS software.

Key Words: Simple Random Sampling, Usual Systematic Sampling, Modified Systematic Sampling, Non-Linear Trend, Average Variances, SAS Software.

1. Introduction

Systematic sampling can be treated as stratified sampling consisting of *n* strata, each stratum consisting of *k* units and we select one unit from each stratum which is located at the same relative position in each stratum. Systematic sampling is also equivalent to cluster sampling by selecting one of the clusters consisting of n units. We select each of the clusters at random. The random start i corresponds i^{th} cluster which consists of the units with labels i+ (j-1)k; j = 1,2,3, …, n.

Let $\bar{y}_{ran}, \bar{y}_{sys}$ and $\bar{y}_{mod}$ be the estimators of population mean $\bar{Y}_N$ under simple random sampling, systematic sampling and modified systematic sampling respectively. The corresponding variance formulae of the estimators for the finite population are known to be

$$\sigma^2_{\bar{y}_{ran}} = \frac{N-n}{Nn}\frac{1}{(N-1)}\sum_{i=1}^{N}(y_i - \bar{Y}_N)^2$$

$$\sigma^2_{\bar{y}_{sys}} = \frac{1}{k}\sum_{i=1}^{k}(\bar{y}_i - \bar{Y}_N)^2$$

$$\sigma^2_{\bar{y}_{mod}} = \frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_i' - \bar{Y}_N\right)^2$$

Where $\bar{y}_i$ and $\bar{y}_i'$ are i[th]sample means under systematic and modified systematic sampling procedures resspectively.

The presence of linear, polynomial or parabolic trend in the population makes the usual systematic sampling less efficient than other sampling schemes. In order to overcome this problem, Cochran (1946) studied the dependence of the performance of systematic sampling on the structure of the population and introduced the so-called super population models. To eliminate the effect of the linear trend, Yates (1948) suggested some end corrections; Sethi (1965) introduced a balanced systematic sampling; Singh et al. (1968) proposed a modified method and Singh and Garg (1979) provided a balanced random sampling. Agarwal and Jain (1988) proposed some improved methods of eliminating linear trend. They also proposed some modified methods of eliminating quadratic trend. Uthayakumaran (1998) claimed that his sampling method is more efficient than circular systematic sampling when the populations exhibit linear and parabolic trends. Modified systematic sampling method of Singh et al. (1968) consists of selecting pair of units equidistant from both ends of the population. The units selected in the i[th] cluster will be

$$i + jk, N - i - jk + 1 \ ; \quad j = 0,1,2,\dots,\frac{n}{2} - 1 \quad for\ n\ even$$

$$i + jk, N - i - jk + 1, i + \frac{1}{2}(n-1)k \ ; \quad j = 0,1,2,\dots,\frac{n}{2} - \frac{3}{2} \quad for\ n\ odd$$

The sample mean is equal to the population mean for n even without applying end corrections. Ashutosh (1995) derived the expressions for the average variances under simple random sampling, systematic sampling and modified systematic sampling for the population having a general trend. The average variances were derived under the condition that random element has no serial correlation, i.e. there is no correlation between the values of e_i and e_j. The average variances with this condition are as follows:

$$E\left(\sigma^2_{\bar{y}_{ran}}\right) = \frac{N-n}{Nn}\frac{1}{(N-1)}E\left[\sum_{i=1}^{N}\{f(i)\}^2 - \frac{1}{N}\{\sum_{i=1}^{N} f(i)\}^2\right] + \frac{N-n}{Nn}\sigma^2 \tag{1}$$

$$E\left(\sigma^2_{\bar{y}_{sys}}\right) = \frac{1}{n^2k}E\left[\sum_{i=1}^{k}\left\{\sum_{j=1}^{n} f(i + \overline{j-1}k)\right\}^2\right] - \frac{1}{N^2}E\{\sum_{i=1}^{N} f(i)\}^2 + \frac{N-n}{Nn}\sigma^2 \tag{2}$$

$$E\left(\sigma^2_{\bar{y}_{mod}}\right) = \frac{1}{n^2k}E\left[\sum_{i=1}^{k}\left\{\sum_{j=0}^{\frac{n}{2}-1}(f(i+jk) + f(N-i-jk+1))\right\}^2\right] - \frac{1}{N^2}\left[\sum_{i=1}^{N} f(i)\right]^2 + \frac{N-n}{Nn}\sigma^2 \tag{3}$$

When n is even and

$$E\left(\sigma^2_{\bar{y}_{mod}}\right) = \frac{1}{n^2k}E\left[\sum_{i=1}^{k}\left\{\sum_{j=0}^{\frac{n}{2}-\frac{3}{2}}\left(f(i+jk)+f(N-i-jk+1)\right)+f\left(i+\frac{n-1}{2}k\right)\right\}^2\right] - \frac{1}{N^2}\left[\sum_{i=1}^{N}f(i)\right]^2 + \frac{N-n}{Nn}\sigma^2 \quad (4)$$

When n is odd.

In the present paper systematic sampling procedures are used for milk yield data exhibiting some non-linear trends. The best fitted mathematical forms of non-linear trend present in the milk yield data are obtained and the expressions of average variances of the estimators of population mean under simple random, usual systematic and modified systematic sampling procedures have been derived for populations showing non-linear trend. A comparative study is made among the three sampling procedures for five data sets by calculating average variances using best fitted trend equations.

2. Average Variances for Non-Linear Trends

Here, we have considered two cases when population have a positively skewed trend characterised by an inverse term model. Using equations (1), (2), (3) and (4), the expressions for average variances of estimates of population mean under simple random sampling, systematic sampling and modified systematic sampling are derived by taking into account the absence of random component i.e. $\sigma^2 = 0$.

Case I

When $f(i)$ is of the form $f(i) = a + bi + \frac{c}{i} + \frac{d}{i^2} + e_i \; ; \; i = 1, 2, 3, \ldots, N$

a, b, c, d are constants, then using (1), we get

$$\sigma^2_{\bar{y}_{ran}} = \frac{N-n}{Nn}\frac{1}{N-1}\left[\sum_{i=1}^{N}\left(a+bi+\frac{c}{i}+\frac{d}{i^2}\right)^2 - \frac{1}{N}\left\{\sum_{i=1}^{N}\left(a+bi+\frac{c}{i}+\frac{d}{i^2}\right)\right\}^2\right]$$

After simplification, we get

$$\sigma^2_{\bar{y}_{ran}} = \frac{N-n}{Nn}\frac{1}{N-1}\left[\frac{N(N^2-1)}{12}b^2 + \left\{2bd - bc(N+1) - 2cd\frac{Q}{N}\right\}P + \{c^2 - \mathrm{bd}(\mathrm{N}+1)\}\,\mathrm{Q} + d^2\mathrm{S} + 2\mathrm{cdR} - \left(\frac{c^2P^2 + d^2Q^2}{N}\right) + 2bcN\right] \quad (5)$$

Where $\sum_{i=1}^{N}\frac{1}{i}=P,\ \ \sum_{i=1}^{N}\frac{1}{i^2}=Q,\ \ \sum_{i=1}^{N}\frac{1}{i^3}=R\ and\ \sum_{i=1}^{N}\frac{1}{i^4}=S$

Similarly using (2) , we get

$$\sigma^2_{\bar{y}_{sys}}=\frac{1}{n^2k}\left[\sum_{i=1}^{k}\left\{\sum_{j=1}^{n}\{a+b(i+\overline{j-1}k)+\frac{c}{(i+\overline{j-1}k)}+\frac{d}{(i+\overline{j-1}k)^2}\}\right\}^2\right]-\frac{1}{N^2}\left[\sum_{i=1}^{N}\left\{a+bi+\frac{c}{i}+\frac{d}{i^2}\right\}\right]^2$$

After simplification, we get the following expression for average variance under systematic sampling

$$\sigma^2_{\bar{y}_{sys}}=\frac{k^2-1}{12}b^2+\frac{1}{N^2n}\left\{N\sum_{i=1}^{k}{P_i}^2-nP^2\right\}c^2+\frac{1}{N^2n}\left\{N\sum_{i=1}^{k}{Q_i}^2-nQ^2\right\}d^2+\frac{2}{N^2n}\left\{N\sum_{i=1}^{k}P_iQ_i-nPQ\right\}cd+\frac{1}{N}\left\{\sum_{i=1}^{k}\{2i+(n-1)k\}P_i-(N+1)P\right\}bc+\frac{1}{N}\left\{\sum_{i=1}^{k}\{2i+(n-1)k\}Q_i-(N+1)Q\right\}bd \tag{6}$$

Where

$\sum_{j=1}^{n}\frac{1}{(i+\overline{j-1}k)}=P_i,\ \ \sum_{j=1}^{n}\frac{1}{(i+\overline{j-1}k)^2}=\ Q_i\ and\ \sum_{i=1}^{k}P_i=\ P\ and\ \sum_{i=1}^{k}Q_i=Q$

Using (3) and (4), we get the following expressions,

When n is even

$$\sigma^2_{\bar{y}_{mod}}=\frac{1}{n^2k}\sum_{i=1}^{k}\left[\sum_{j=0}^{\frac{n}{2}-1}\left\{a+b(i+jk)+\frac{c}{(i+jk)}+\frac{d}{(i+jk)^2}+a+b(N-i-jk+1)+\frac{c}{(N-i-jk+1)}+\frac{d}{(N-i-jk+1)^2}\right\}\right]^2-\frac{1}{N^2}\left[\sum_{i=1}^{N}\left(a+bi+\frac{c}{i}+\frac{d}{i^2}\right)\right]^2$$

Which after simplification reduces to

$$\sigma^2_{\bar{y}_{mod}} = \frac{1}{N^2 n}\left[\left\{N\sum_{i=1}^{k} P_i^2 - nP^2\right\}c^2 + \left\{N\sum_{i=1}^{k} Q_i^2 - nQ^2\right\}d^2 + 2\left\{N\sum_{i=1}^{k} P_i Q_i - nPQ\right\}cd\right] \quad (7)$$

Where

$$\sum_{j=0}^{\frac{n}{2}-1}\left(\frac{1}{i+jk} + \frac{1}{n-i-jk+1}\right) = P_i \ and \ \sum_{j=0}^{\frac{n}{2}-1}\left(\frac{1}{i+jk} + \frac{1}{n-i-jk+1}\right) = Q_i \ such\ that \sum_{i=1}^{k} P_i = P \ and \ \sum_{i=1}^{k} Q_i = Q$$

When n is odd

$$\sigma^2_{\bar{y}_{mod}} = \frac{1}{n^2 k}\sum_{i=1}^{k}\left[\sum_{j=0}^{\frac{n}{2}-1}\left\{a + b(i+jk) + \frac{c}{(i+jk)} + \frac{d}{(i+jk)^2} + a + b(N-i-jk+1) + \frac{c}{(N-i-jk+1)} + \frac{d}{(N-i-jk+1)^2}\right\} + a + b\left(i + \frac{(n-1)}{2}k\right) + \frac{c}{\left(i + \frac{(n-1)}{2}k\right)} + \frac{d}{\left(i + \frac{(n-1)}{2}k\right)^2}\right]^2 - \frac{1}{N^2}\left[\sum_{i=1}^{N}\left(a + bi + \frac{c}{i} + \frac{d}{i^2}\right)\right]^2$$

Which after simplification reduces to

$$\sigma^2_{\bar{y}_{mod}} = \frac{(k^2-1)}{12n^2}b^2 + \frac{1}{N^2 n}\left[\left\{N\sum_{i=1}^{k} {P_i}^2 - nP^2\right\}c^2 + \left\{N\sum_{i=1}^{k} {Q_i}^2 - nQ^2\right\}d^2 + 2\left\{N\sum_{i=1}^{k} P_i Q_i - nPQ\right\}cd\right] + \frac{1}{Nn}\left[2\sum_{i=1}^{k}\left\{i + \frac{(n-1)}{2}k + \frac{(n-1)(N+1)}{2}\right\}P_i - n(N+1)P\right]bc + \frac{1}{Nn}\left[2\sum_{i=1}^{k}\left\{i + \frac{(n-1)}{2}k + \frac{(n-1)(N+1)}{2}\right\}Q_i - n(N+1)Q\right]bd \quad (8)$$

Where

$$Q_i = \left[\sum_{j=0}^{\frac{n}{2}-\frac{3}{2}}\left(\frac{1}{(i+jk)^2}+\frac{1}{(N-i-jk+1)^2}\right)+\frac{1}{\left(i+\frac{(n-1)}{2}k\right)^2}\right] \; such\ that\ \sum_{i=1}^{k} P_i = P$$

$$and \sum_{i=1}^{k} Q_i = Q$$

Case II

When $f(i)$ is of the form

$$f(i) = a + bi + ci^2 + \frac{d}{i} + e_i\ ;\ i = 1,2,3,\dots,N$$

Then expressions for average variances under different sampling procedures are given by

$$\sigma^2_{\bar{y}_{ran}} = \frac{N-n}{Nn}\frac{1}{N-1}\left[\frac{N(N^2-1)}{12}b^2 + \frac{N(N^2-1)(2N+1)(8N+11)}{180}c^2 + \left\{Q-\frac{P^2}{N}\right\}d^2 + \frac{N(N+1)^2(N-1)}{6}bc + \{2N-(N+1)P\}bd + \{3N-(2N+1)P\}\frac{(N+1)}{3}cd\right] \tag{9}$$

$$\sigma^2_{\bar{y}_{sys}} = \frac{k^2-1}{12}b^2 + \left\{\left(\frac{k^4}{5}+\frac{k^3}{2}+\frac{k^2}{3}-\frac{1}{30}\right) + k(k+1)(n-1)\left(\frac{nk^2}{3}+\frac{nk}{6}+\frac{k^2}{6}+\frac{k}{3}\right) - (k+1)^2\left(\frac{nk}{2}-\frac{k}{6}+\frac{1}{6}\right)^2\right\}c^2 + \frac{1}{N^2n}\left\{N\sum_{i=1}^{k}P_i^2 - nP^2\right\}d^2 + \frac{(k^2-1)(nk+1)}{6}bc + \frac{1}{N}\left\{\sum_{i=1}^{k}\{2i+(n-1)k\}P_i - (N+1)P\right\}bd + \frac{1}{3N}\left\{\sum_{i=1}^{k}\{6i^2+(2n^2-3n+1)k^2+6in(n-1)k\}P_i - (N+1)(2N+1)P\right\}cd \tag{10}$$

$$\sigma^2_{\bar{y}_{mod}} = \left\{\left(\frac{k^4}{5}+\frac{k^3}{2}+\frac{k^2}{3}-\frac{1}{30}\right)-(k+1)^2\left(\frac{nk}{4}+\frac{k}{6}+\frac{1}{3}\right)^2 + (k+1)\left(\frac{nk}{2}+k+1\right)\left(\frac{nk^2}{6}+\frac{nk}{12}-\frac{k^2}{6}+\frac{1}{6}\right)\right\}c^2 + \frac{1}{N^2n}\left[N\sum_{i=1}^{k}P_i^2 - nP^2\right]d^2 + \frac{2}{N^2n}\left[N\left\{\sum_{i=1}^{k}\left(\sum_{j=0}^{\frac{n}{2}-1}Z_{ij}\right)P_i\right\} - \frac{nN(N+1)(2N+1)}{6}P\right]cd \tag{11}$$

$$Where\ P=\sum_{i=1}^{N}\frac{1}{i}, Q=\sum_{i=1}^{N}\frac{1}{i^2}, R=\sum_{i=1}^{N}\frac{1}{i^3}, S=\sum_{i=1}^{N}\frac{1}{i^4}, P_i=\sum_{j=1}^{n}\frac{1}{(i+\overline{j-1}k)},$$

$$Q_i=\sum_{j=1}^{n}\frac{1}{(i+\overline{j-1}k)^2}\ and$$

$$Z_{ij} = \{(2i^2+2j^2k^2+4ijk)-2i(N+1)-2jk(N+1)+(N+1)^2\}$$

Case 3

A comparison of systematic sampling and modified systematic sampling procedures when the trend function is quadratic may be useful. If we have,

$$f(i) = a + bi + ci^2 + e_i \tag{12}$$

In the absence of random component the average variances under random sampling, usual systematic sampling and modified systematic sampling derived by Ashutosh (1995) and Singh et al (1968) are given by the following formulae:

$$E\left(\sigma^2_{\bar{y}_{ran}}\right) = \frac{(nk+1)(k-1)}{12}b^2 + \frac{(nk+1)(2nk+1)(8nk+11)(k-1)}{180}c^2 + \frac{(nk+1)^2(k-1)}{6}bc \tag{13}$$

$$E\left(\sigma^2_{\bar{y}_{sys}}\right) = \frac{(k^2-1)}{12}b^2 + \left\{\left(\frac{k^4}{5}+\frac{k^3}{2}+\frac{k^2}{3}-\frac{1}{30}\right)+k(k+1)(n-1)\left(\frac{nk^2}{3}+\frac{nk}{6}+\frac{k^2}{6}+\frac{k}{3}\right) - (k+1)^2\left(\frac{nk}{2}-\frac{k}{6}+\frac{1}{6}\right)^2\right\}c^2 + \frac{(k^2-1)(nk+1)}{6}bc \tag{14}$$

$$E\left(\sigma^2_{\bar{y}_{mod}}\right) = \left\{\left(\frac{k^4}{5} + \frac{k^3}{2} + \frac{k^2}{3} - \frac{1}{30}\right) - (k+1)^2\left(\frac{nk}{4} + \frac{k}{6} + \frac{1}{3}\right)^2 + (k+1)\left(\frac{nk}{2} + k + 1\right)\left(\frac{nk^2}{6} + \frac{nk}{12} - \frac{k^2}{6} + \frac{1}{6}\right)\right\} c^2 \tag{15}$$

3. Illustrations

The milk yield data used for the study have been collected from Instructional Dairy Farm of G.B. Pant University of Agriculture and Technology, Pantnagar. The data relate to milk yields of four breeds of cows (two brands S16 and S19 of Sahiwal cows and two brands X124 and X205 of crossbred cows) and one breed (Murrah brand No. M125) of buffaloes over one lactation period during 2011-2013. These breeds are chosen because the lactation records show mostly the assumed non-linear trends. The records are in Litre units. For simplicity, we have not taken into account yield records during the colostrum period. Tables 1, 2, 3, 4 and 5 show the milk yield data for the abovesaid cows and buffaloes.

5	6	6	5	6	7	8	9	6	8	8	8	9	8	8	8
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
8	8	8	8	7	7	8	7	6	6	6	7	7	8	8	7
7	8	8	7	8	8	7	7	8	7	8	8	7	8	8	7
8	8	8	8	8	8	8	7	9	9	9	10	9	8	8	8
8	9	8	8	8	9	9	8	9	10	10	10	9	9	10	8
10	10	10	10	10	10	10	11	10	11	9	10	10	9	10	9
10	8	9	8	9	9	9	10	9	8	8	8	8	9	9	9
9	9	8	8	8	8	7	8	9	9	9	8	8	9	9	8
9	9	9	9	8	10	8	8	9	8	9	10	9	9	10	8
9	10	7	8	9	9	9	8	8	8	8	8	8	6	7	7
7	6	6	7	7	8	6	7	8	7	6	8	8	8	8	8
5	6	6	5	4	4	4	4	4	4	4	4	5	7	6	5
6	7	6	5	5	6	4	4	5	5	4	4	4	4	4	4
5	5	5	6	5	6	4	5	4	5	4	4	3	4	4	3
5	5	5	5	4	4	4	4	4	3	5	4	1	1	1	1

Table 1: Horizontally Day wise Milk Yield Data (in Litres) for S-16 brand of Sahiwal Cows for 256 days from the date (10/03/2011) of calving

11	10	10	10	11	10	10	10	10	9	8	11	11	12	12	11
12	13	10	11	11	11	12	12	9	8	4	10	10	12	12	12
14	10	10	10	11	11	10	11	12	12	11	12	12	10	11	10
11	11	10	11	11	11	10	10	11	11	11	10	10	10	10	12
12	12	11	10	11	12	11	11	10	10	10	10	10	11	10	12
10	10	11	11	11	11	11	10	12	11	11	11	11	11	10	10
10	10	11	11	11	12	12	12	10	11	12	10	8	9	10	11
10	10	10	10	10	9	9	9	8	8	8	8	9	10	10	9
9	8	8	9	10	9	9	8	8	8	10	11	9	9	10	9
10	9	9	8	9	8	8	9	9	9	8	9	9	10	8	10
8	8	10	9	10	10	10	10	9	10	9	10	8	9	9	8
9	9	9	8	8	8	8	8	8	8	6	6	6	7	8	5
8	8	8	8	8	8	8	8	9	8	8	8	8	7	6	6
6	6	6	5	6	6	7	4	5	5	6	8	6	7	7	6
5	6	6	5	6	5	4	5	4	4	5	5	5	4	6	5
4	5	4	4	3	4	3	5	5	2	0	1				

Table 2: Horizontally Day wise Milk Yield Data (in Litres) for S-19 brand of Sahiwal Cows for 252 days from the date (26/02/2011) of calving

10	10	10	10	10	10	10	10	10	10	10	10	11	12	12	12
12	12	12	12	12	12	12	13	12	12	12	12	12	12	13	11
12	12	12	12	14	13	13	13	13	13	13	12	12	12	12	13
13	12	12	13	12	12	12	12	13	12	12	12	12	12	12	12
12	12	12	12	13	12	12	12	12	12	12	12	12	13	12	12
12	12	13	12	13	12	12	12	12	12	12	12	12	12	12	12
12	11	10	14	15	14	14	13	14	14	12	14	14	14	13	14
12	14	13	14	14	14	13	14	14	14	13	14	13	12	13	13
12	12	12	12	12	12	13	13	15	12	13	12	12	12	12	12
11	11	12	12	12	12	12	12	12	12	13	12	12	12	12	12
12	12	12	12	12	13	12	13	11	11	11	10	10	13	13	12
13	12	12	12	12	13	12	12	13	12	12	12	11	10	11	11
11	11	11	10	10	10	11	11	10	10	10	12	12	12	12	12
12	12	12	13	13	13	14	15	14	14	14	12	14	14	14	12
12	12	12	12	10	11	9	8	8	8	8	8	8	9	8	8
8	8	8	8	8	9	8	8	8	9	8	9	8	8	8	8
8	8	8	8	8	8	8	9	9	9	9	9	9	8	8	8
8	9	8	3	2	2	1	0								

Table 3: Horizontally Day wise Milk Yield Data (in Litres) for X-124 brand of Crossbred Cows for 280 days from the date (15/07/2012) of calving

8	10	8	8	17	18	18	18	18	18	18	18	18	20	24	24
24	26	26	26	26	24	25	26	27	26	27	26	24	24	25	26
25	25	22	21	26	24	24	24	26	26	27	27	27	28	28	27
26	15	25	27	27	26	26	27	27	28	28	26	26	27	26	26
27	28	27	26	28	17	25	27	27	27	29	26	26	26	26	27
25	27	22	25	25	24	24	24	24	23	22	23	24	24	23	22
21	23	23	24	23	23	23	23	22	23	23	22	22	23	23	24
21	20	20	21	21	21	20	20	20	20	21	20	20	20	18	19
19	18	20	20	21	20	21	19	18	19	18	19	19	18	19	18
21	21	21	22	22	20	20	21	22	21	20	20	21	20	20	21
20	22	22	20	19	20	20	20	21	21	20	20	20	21	21	17
19	18	18	19	18	18	17	16	17	19	20	21	20	21	20	20
20	20	19	19	18	19	20	20	18	16	16	17	16	16	17	14
15	16	16	16	16	15	16	16	17	16	15	14	15	15	16	15
16	14	13	12	13	14	13	12	12	11	11	10	16	16	17	17
15	14	15	15	15	15	17	17	18	17	16	16	17	16	16	16
17	16	16	17	17	16	16	16	16	16	17	13	13	13	12	12
12	12	12	13	12	13	12	13	12	12	13	12	12	12	12	14
13	12	12	13	12	12	13	12	13	13	13	14	13	12	12	14
14	14	12	12	12	13	13	12	12	13	13	12	13	12	11	12
10	10	11	11	11	11	11	12	12	12	12	13	12	12	12	12
12	11	13	12	12	12	11	11	11	10	10	11	11	12	11	10
10	11	10	11	11	11	10	12	12	12	12	13	12	12	12	13
12	12	12	12	12	12	12	12	12	12	12	12	13	11	12	11
12	12	12	11	11	14	12	12	12	10	12	12	12	13	12	12
12	12	13	13	12	12	10	11	12	8	8	7	6	7	6	7
6	5	4	1												

Table 4: Horizontally Day wise Milk Yield Data (in Litres) for X-205 brand of Crossbred Cows for 420 days from the date (12/03/2011) of calving

4	5	4	4	4	6	6	6	7	7	9	9	8	8	8	8
8	9	10	9	9	10	10	10	9	9	10	10	9	8	9	9
9	9	9	9	9	9	8	9	9	5	9	4	10	9	10	11
12	11	10	10	5	5	4	4	5	5	5	5	6	6	7	7
7	6	6	7	7	7	7	6	6	6	5	5	6	6	7	6
6	6	6	6	6	6	6	6	6	6	6	7	6	6	6	7
6	7	6	6	6	5	5	5	5	5	5	5	5	5	4	5
5	5	5	5	5	5	5	5	5	5	5	5	4	4	4	4
5	4	4	4	4	4	4	4	4	4	4	4	4	5	5	5
5	6	5	5	6	6	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	4	4	4	4	4	4	4	4	5	5
4	5	5	6	5	6	5	6	4	4	4	3	4	4	4	5
5	6	4	3	3	3	4	4	3	3	4	3	4	3	4	3

3	3	5	3	3	3	3	3	3	3	3	3	4	3	3	4
4	3	4	3	3	3	3	3	3	2	3	3	2	4	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	0
4	4	4	4	5	5	5	4								

Table 5: Horizontally Day wise Milk Yield Data (in Litres) for M-125 brand of Murrah Buffaloes for 264 days from the date (22/07/2012) of calving

4. Results and Discussion

(i) For data set I (Sahiwal Cows Brand No. S16), the best fitted trend equation is

$$y_i = 6.3712 + 0.0508i - 0.00026i^2 - \frac{0.000000148}{i}$$

with the highest value (0.7545) of R^2 and minimum value (248.3393) of S.S.R. In this case, usual systematic sampling is better with minimum average variance (0.0013) than simple random and modified systematic sampling. Moreover, Sahiwal Cows Brand No. S16 give milk yield of 7.2 Litres/day on an average during its lactation period of 256 days.

(ii) For data set II (Sahiwal Cows Brand No. S19), the best fitted trend equation is

$$y_i = 10.1781 + 0.0213i - 0.00018i^2 + \frac{0.4029}{i}$$

with the highest value (0.7951) of R^2 and minimum value (299.5241) of S.S.R. In this case, modified systematic sampling (when n = 28 and k = 9) is better with minimum average variance (0.0034) than simple random and usual systematic sampling. Moreover, Sahiwal Cows Brand No. S19 give milk yield of 8.39 Litres/day on an average during its lactation period of 252 days.

(iii) For data set III (Crossbred Cows Brand No. X124), the best fitted trend equation is

$$y_i = 10.3585 + 0.0484i - 0.00022i^2 - \frac{0.6448}{i}$$

with the highest value (0.6407) of $R^2$and minimum value (473.421) of S.S.R. In this case, usual systematic sampling (when n = 20 and k = 14) is better with minimum average variance (0.0026) than simple random and modified systematic sampling. Moreover, Crossbred Cows Brand No. X124 give milk yield of 11.29 Litres/day on an average during its lactation period of 280 days. Further, it is also found that by reducing sampling interval of 14 days to 10 days, efficiency of usual systematic sampling scheme increases, whereas the efficiency of modified systematic sampling decreases. Thus, it may be concluded that for data set III, it may be better to use sampling interval of 10 days rather than 14 days.

(iv) For data set IV (Crossbred Cows Brand No. X205), the best fitted trend equation is

$$y_i = 28.0716 - 0.0479i - \frac{71.0201}{i} + \frac{51.5425}{i^2}$$

with the highest value (0.8595) of $R^2$and minimum value (1841.349) of S.S.R. In this case, usual systematic sampling is better with minimum average variance (0.0055) than simple random and modified systematic sampling. Moreover, Crossbred Cows Brand No. X205 give milk yield of 17.05 Litres/day on an average during its lactation period of 420 days.

(v) For data set V (Murrah Buffaloes Brand No. M125), the best fitted trend equation is

$$y_i = 9.5595 - 0.0454i + 0.000081i^2 - \frac{8.3945}{i}$$

with the highest value (0.6872) of R^2 and minimum value (344.7382) of S.S.R. In this case, usual systematic sampling is better with minimum average variance (0.0036) than simple random and modified systematic sampling. Moreover, Murrah Buffaloes Brand No. M125 give milk yield of 5.24 Litres/day on an average during its lactation period of 264 days.

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