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RELIABILITY APPROXIMATION FOR SOLID SHAFT UNDER GAMMA SETUP

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Abstract

This paper proposes a new approach for evaluating the reliability ofsystem where stress and strength are defined as complex function and whose reliability is not derivable through analytic techniques. The discretization was the earlier approach for reliability approximation. But this method fails to provide extent of error in terms of distributional parameters. This proposes method of offering bound based approach where reliability planner`s not only get a clear idea about the extent of error but also can manipulate in terms of design parameters.Here, this reliability approximation has been under taken under the Gamma setup which is widely used model for reliability analysis. Using our work, reliability planners will be able to obtain reliability in terms of design parameters during the early stages of product design and adjust it according to their requirements.

Key Words: System Reliability, Reliability bounds, Reliability Approximation and Extent of Error.

Notations

IFR	= Increasing failure rate
$F_x(X)$	= Corresponding cumulative distribution (df) function.
G (λ, P)	= Gamma distribution with scale parameter λ and shape parameter P.
E(X)	=Expectation on random variable X.
R	= System reliability
U (λ_A, $\lambda_{M,}$λ, P, a)	= Reliability upper bound
L (λ_A, $\lambda_{M,}$λ, P)	= Reliability lower bound
R_{approx}	= Reliability approximation

1. Introduction

In the context of reliability analysis of mechanical engineering, the stress-strength model has been widely used. In this model, reliability is defined as P(X>Y), where X is the strength and Y is the stress i.e. the probability that strength exceed the stress.

Since the form of the stress function is generally complicated in nature, finding an exact distribution of the same is intractable in most of the cases. We observe from the literature that some alternative approaches are available for approximating the system reliability. These are (i) Taylor-series method (ii) Monte-Carlo method (iii) Quadrature method (iv) Discretization Method and (v) Discrete approximation method. (vi) Reliability approximation technique. Evans (1975) has reviewed the first three methods with their relative advantages and disadvantages. The concept of discretization was imbedded in factorial experiment method, proposed by Taguchi (1978) to approximate a normal distribution by a 3-point discrete distribution. Recently, reliability approximation based on reliability bounds under the stress strength model has been examined by Nayak (2011). Nayak and Roy (2012) have proposed bound based reliability approximation under the Weibull, Rayleigh and Exponential setups. Recently, Nayak and Roy (2012) also have studied a new approach for approximating reliability of a complex system under the Weibull setup. Recently, Nayak and Roy (2012) have proposed a new approach for approximating reliability of an engineering item, resistor, under the Weibull setup. Approximation of system reliability has been studied by Xie and Lai (1998) using one step conditioning. A stress-strength inference reliability model with strength degradation under the assumptions that stress-strength are statistically independent have been examined by Xue and Yang (1997). They have also presented simple formulas for estimating upper and lower bounds for stress-strength reliability Here the case where X and Y are independent Gamma random variables is considered.

In further investigation regarding the stress- strength model, it has become an important research topic to calculate system reliability based on probability distribution of stress and strength. Park et al. (1986) considered computational algorithm for reliability bounds in probabilistic design. Another empirical approach was proposed by Shen (1992) to calculate unreliability bounds based upon the subinterval probabilities of stress and strength in the inference region. An algorithm for computing the unreliability bounds based on an improved Monte Carlo method was proposed by Guo et al (1994).

Determination of reliability approximation of complex system based on reliability bounds is the main purpose of this paper. Interesting feature of the proposed approach is that the reliability approximation comes out as a function of design parameters and it can be adjusted for designing and redesigning the system to ensure the maximum level of reliability at a given cost as per requirements. In section 2 reliability bounds are obtained. Section 3 gives extent of error in terms of two bounds and also gives the approximate reliability, where as section 4 gives a numerical study of this proposed reliability and error.

2. Reliability bounds

Reliability bounds are item dependent and setup dependent. Here, a very common but an important engineering item for evaluating the reliability bounds under Gamma setup is studied. It was seen from the literature that different authors assumed stress and strength factors to be normal or at least unimodal and symmetric for the assessment of reliability in complex system .Yet the normal setup may sometimes fails in practical application, especially because of the tails of its distribution. Other parametric families can be used for modelling stress and strength components. Here two parameter Gamma family is taken, as this is able to model a great variety of density functions by varying its parameters, like Weibull or Beta. Therefore, a study on reliability approximation based on reliability bounds has wider appeal under gamma setup.

Solid shaft is an engineering item of importance to design engineers (Singer and Pytel, 1980). The shear stress of it can be expressed in the following functional form i.e.

f(M,A) $=\frac{16M}{\pi A^3}$

Where M= Torque placed upon it, A= Outer diameter of the Shaft.

Under this it is assumed that M follows G (λ_M, P) and A follows G (λ_A, P). It is also assumed that strength random variable X follows G (λ,P). Another assumption is that M, A, and X are mutually independent. Under this, problem is to evaluate system reliability, R. Now two lemma`s to reach upper and lower reliability bounds on R are stated.

Lemma1: If X is random variable such that $E(e^{ax})$ exists for a > 0 then $P(X > t) \leq \frac{E(e^{ax})}{e^{at}}$

Lemma2: If F∈ IFR and E (T) = μ then for 0≤ t ≤μ,$R \geq \exp(-\frac{t}{\mu})$

Result 1: If the strength and the stress component random variables of the Solid-shaft follow Gamma distributions then upper bound for the system reliability, R, is given by

U $(\lambda_A, \lambda_M, \lambda$, P, a) $=(\frac{\lambda}{\lambda-a})^P\{1-\frac{16aP\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}+\frac{128a^2\Gamma(P+2)\Gamma(P-6)\lambda_A^6}{\pi^2\{\Gamma(P)\}^2\lambda_M^2}\}$, for P> 6

Proof: According to the definition of reliability

R=P(X> Y) , where Y = $\frac{16M}{\pi A^3}$

Using lemma1, we get

$$R=P(X> y_{|Y=y}) \leq \frac{E(e^{aX})}{e^{ay}}$$

$$=E_A E_M\left[\frac{\left(\frac{\lambda}{\lambda-a}\right)^P}{e^{a\frac{16M}{\pi A^3}}}\right]$$

$$=\int_0^\infty \int_0^\infty (\frac{\lambda}{\lambda-a})^P e^{-a\frac{16M}{\pi A^3}} dF_M dF_A \quad (1)$$

Note that $e^{-X} \leq 1 - X + \frac{X^2}{2}$,for X>0

Therefore (1) reduces to

$$R \leq (\frac{\lambda}{\lambda-a})^P \int_0^\infty \int_0^\infty \{1-\frac{16aM}{\pi A^3}+\frac{(16aM)^2}{2A^6\pi^2}\}\, dF_M\, dF_A$$

$$= (\frac{\lambda}{\lambda-a})^P \{1-\frac{16aE(M)E\left(\frac{1}{A^3}\right)}{\pi}+\frac{128a^2E(M^2)E(\frac{1}{A^6})}{\pi^2}\}$$

$$= (\frac{\lambda}{\lambda-a})^P \{1-\frac{16aP\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}+\frac{128a^2\Gamma(P+2)\Gamma(P-6)\lambda_A^6}{\pi^2\{\Gamma(P)\}^2\lambda_M^2}\}, \text{ for } P>6 \quad (2)$$

Hence, upper bound for the system reliability, R, of the Solid-shaft under the Gamma setup is given by

$$U(\lambda_A, \lambda_M, \lambda, P, a) = (\frac{\lambda}{\lambda-a})^P \{1-\frac{16aP\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}+\frac{128a^2\Gamma(P+2)\Gamma(P-6)\lambda_A^6}{\pi^2\{\Gamma(P)\}^2\lambda_M^2}\}, \text{ for } P>6$$

This completes the proof.

Result 2: If the strength and the stress component random variable of the Solid-shaft follows Gamma distribution then lower bound for the system reliability, R, is given by

$$L(\lambda_A, \lambda_M, \lambda, P) = 1-\frac{16\lambda\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}, \text{ for } P>3$$

Proof: Using lemma2, $R \geq \exp\left(-\frac{Y}{\mu_x}\right)$

Therefore,

$$R \geq \exp(-\frac{\lambda Y}{P})$$

$$= exp(-\frac{\lambda\frac{16M}{\pi A^3}}{P})$$

$$= E_A E_M (e^{-\frac{16M\lambda}{\pi A^3 P}})$$

$$= \int_0^\infty \int_0^\infty (e^{-\frac{16M\lambda}{\pi A^3 P}}) dF_M\, dF_A \quad (3)$$

Note that $e^{-X} \geq 1-X$, for X>0

Therefore (3) reduces to

$$= \int_0^\infty \int_0^\infty [1-\frac{\lambda}{P}\frac{16M}{\pi A^3}]\, dF_M\, dF_A$$

$$= 1-\frac{16\lambda E(M)E(\frac{1}{A^3})}{P\pi}$$

$$=1-\frac{16\lambda\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}, \text{ for } P>3 \tag{4}$$

Hence, Lower bound for the system reliability, R, of the Solid-shaft under the Gamma setup is given by

$$L(\lambda_A, \lambda_M, \lambda, P) = 1-\frac{16\lambda\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}, \text{ for } P>3$$

3. Reliability approximation and Extent of Error

Here average of two bounds is proposed as the reliability approximation and half of the absolute difference between the two bounds as the extent of error. It is obtained in terms of distributional parameters. Reliability approximation is given by

$$R_{approx} = \frac{U(\lambda_A, \lambda_M, \lambda, P, a) + L(\lambda_A, \lambda_M, \lambda, P)}{2}$$

$$=\frac{\left(\frac{\lambda}{\lambda-a}\right)^P\left\{1-\frac{16aP\,\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}+\frac{128a^2\Gamma(P+2)\Gamma(P-6)\lambda_A^6}{\pi^2\{\Gamma(P)\}^2\lambda_M^2}\right\}+\left\{1-\frac{16\lambda\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}\right\}}{2} \tag{5}$$

Extent of error is given by

$$\text{Error} \leq \frac{U(\lambda_A, \lambda_M, \lambda, P, a) - L(\lambda_A, \lambda_M, \lambda, P)}{2}$$

$$=\frac{\left(\frac{\lambda}{\lambda-a}\right)^P\left\{1-\frac{16aP\,\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}+\frac{128a^2\Gamma(P+2)\Gamma(P-6)\lambda_A^6}{\pi^2\{\Gamma(P)\}^2\lambda_M^2}\right\}-\left\{1-\frac{16\lambda\Gamma(P-3)\lambda_A^3}{\pi\Gamma(P)\lambda_M}\right\}}{2} \tag{6}$$

4. Numerical Study

A numerical study of the reliability approximation along with the extent of error is done in this section using equations (5) & (6). For this purpose, lower and upper reliability bounds for some specific choices of the distributional parameters of the Solid-shaft are calculated. The specific choices of distributional parameters, considered here, are P=90, a=.00001, $\lambda = 2500$, $\lambda_A = 3000$ and the other parameter, λ_M, is allowed to vary so that it covers a wide range of reliability values. The corresponding reliability approximation and extent of error are shown in the table1. It may be observed from the given table that error term sharply decreases as reliability increases.

Sl.No.	λ_M	Reliability approximation	Extent of error
1	3.00E+09	0.915947	0.084054
2	3.50E+09	0.927954	0.072046
3	4.00E+09	0.936960	0.063040
4	5.00E+09	0.949568	0.050432

5	6.00E+09	0.957973	0.042027
6	7.00E+09	0.963977	0.036023
7	8.00E+09	0.968480	0.031520
8	9.00E+09	0.971982	0.028018
9	1.00E+10	0.974784	0.025216
10	2.00E+10	0.987392	0.012608
11	3.00E+10	0.991595	0.008406
12	4.00E+10	0.993696	0.006304
13	5.00E+10	0.994957	0.005043
14	6.00E+10	0.995798	0.004203
15	7.00E+10	0.996398	0.003602
16	9.00E+10	0.997198	0.002802
17	1.00E+11	0.997479	0.002522
18	2.00E+11	0.998739	0.001261
19	3.00E+12	0.999916	8.42E-05

Table 1: Reliability approximation and Extent of error under the proposed method.

5. Conclusion

A method for approximating the reliability of the complex system where exact determination of reliability is not tractable has been suggested .In this paper, a bound based reliability approximation technique for Gamma setup under the stress- strength model is presented. This work has greater applicability as it has been presented in terms of design parameters and can be adjusted as per requirements. Therefore, under the stress-strength model, this method for approximating the system reliability for intractable cases is suggested. There are cases where discrete approximations are extremely weak. For example, under the exponential setup, where lack of memory property holds, the discretization approach does not offer close approximate values. In that situation, one may depend on bound based reliability approximation.

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