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DUAL TO RATIO AND PRODUCT TYPE EXPONENTIAL ESTIMATORS IN STRATIFIED RANDOM SAMPLING USING TWO AUXILIARY VARIATES

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Abstract

This paper discusses the problem of estimation of population mean in stratified random sampling. In fact, in this paper, dual to ratio and product type exponential estimators in stratified random sampling have been suggested. The biases and mean squared errors of the suggested estimators haven been obtained up to the first degree of approximation. The suggested estimators have been compared with existing estimators and conditions under which suggested estimators are more efficient than other considered estimators have been obtained. An empirical study has been carried out to demonstrate the performance of the suggested estimators.

Key Words: Exponential Estimator, Stratified Random Sampling, Bias and Mean Squared Error.

1. Introduction

Cochran (1940) utilized auxiliary information at estimation stage and suggested ratio method of estimation. The ratio estimator is more efficient when study variate and auxiliary variate are positively correlated. Robson (1957) developed product method of estimation that provides product estimator is more efficient than the simple mean estimator when the study variate and auxiliary variate are negatively correlated.
Searl (1964) utilized coefficient of variation of the study variate for the estimation of parameters of the study variate. Sisodia and Dwivedi (1981) suggested ratio type estimator for population means using coefficient of variation of auxiliary variate . Later many authors including Pandey and Dubey (1988), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et al. (2004), Kadilar and Cingi (2006) used various known parameters of auxiliary variate such as coefficient of variation, coefficient of kurtosis and correlation coefficient between study variate and auxiliary variate. Bahl and Tuteja (1991) used an exponential function and defined ratio and product type exponential estimators using population mean of auxiliary variate.

Srivenkataramana (1980) and Bandhyopadhyaya (1980) applied a transformation on auxiliary variate and developed dual to classical ratio and product estimators independently.

Work cited above is more concerned with simple random sampling. Since simple random sampling has its limitations that it is suitable when population is homogeneous. When population is heterogeneous stratified random sampling is used in which whole population is divided into homogeneous gropes called strata and a sample of predetermined size is drawn from each stratum. Stratified random sampling is also useful when estimates of sub-groups are also required.

Hansen et al. (1946) defined combined ratio estimator in stratified random sampling. Kadilar and Cingi (2003) used various known parameters of auxiliary variate such as coefficient of variation and coefficient of kurtosis and studied various ratio type estimators in stratified random sampling.

Singh et al. (2008) defined Bahl and Tuteja (1991) ratio and product type exponential estimators in stratified random sampling. A generalized ratio-cum-product estimator of finite population mean in stratified random sampling was discussed by Tailor et al. (2011). Recently, Chouhan (2012) studied many problems of estimation of population mean in stratified random sampling. In this paper, dual to Singh et al. (2008) estimators are suggested with its properties.

Consider a finite population $U = U_1, U_2, ..., U_N$ of size N and it is divided into L strata of size N_h $(h = 1,2,..., L)$. Let y be the study variate and x and z be two auxiliary variates taking values y_{hi}, x_{hi} and z_{hi} $(h = 1,2,..., L;\ i = 1, 2,...., N_h)$, on i^{th} unit of the h^{th} stratum . A sample of size n_h is drawn from each stratum which constitutes a sample of size $n = \sum_{h=1}^{L} n_h$ then we define

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum mean for the study variate y ,

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for the auxiliary variate x,

$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} z_{hi}$: h^{th} stratum mean for the auxiliary variate z,

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variate y ,

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of the auxiliary variate x,

$\bar{Z} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{Z}_h$: Population mean of the auxiliary variate z,

$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} \bar{y}_{hi}$: Sample mean of the study variate y for h^{th} stratum,

$\bar{x}_h = \dfrac{1}{n_h}\sum_{i=1}^{n_h} \bar{x}_{hi}$: Sample mean of the auxiliary variate *x* for h^{th} stratum,

$\bar{z}_h = \dfrac{1}{n_h}\sum_{i=1}^{n_h} \bar{z}_{hi}$: Sample mean of the auxiliary variate *z* for h^{th} stratum,

$W_h = \dfrac{N_h}{N}$: Stratum weight of h^{th} stratum.

Classical ratio and product estimators for estimating the population mean $\bar{Y}$ are defined as

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.1}$$

$$\hat{\bar{Y}}_P = \bar{y}\left(\frac{\bar{z}}{\bar{Z}}\right) \tag{1.2}$$

where $\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{z} = \dfrac{1}{n}\sum_{i=1}^{n} z_i$ are sample means the auxiliary variates x and z respectively .

Srivenkatramana (1980) utilized a transformation $x_i^* = \dfrac{N\bar{X} - nx_i}{N-n}$ and suggested dual to classical ratio and product estimators for estimating the population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_R^* = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right), \tag{1.3}$$

$$\hat{\bar{Y}}_P^* = \bar{y}\left(\frac{\bar{Z}}{\bar{z}^*}\right) \tag{1.4}$$

where

$$\bar{x}^* = \frac{\bar{X}N - \bar{x}n}{N-n} \quad \text{and} \quad \bar{z}^* = \frac{\bar{Z}N - \bar{z}n}{N-n}.$$

Hansen et al. (1946) defined combined ratio estimator for estimating the population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{Rc} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{1.5}$$

Combined product estimator for estimating the population mean $\bar{Y}$ can be defined as

$$\hat{\bar{Y}}_{Pc} = \bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \quad (1.6)$$

Kushwaha et al.(1990) obtained dual to combined ratio and product estimators $\hat{\bar{Y}}_{RC}$ *and* $\hat{\bar{Y}}_{PC}$ respectively as

$$\hat{\bar{Y}}^{*}_{RC} = \bar{y}_{st}\left(\frac{\bar{x}^{*}_{st}}{\bar{X}}\right) \quad (1.7)$$

$$\hat{\bar{Y}}^{*}_{PC} = \bar{y}_{st}\left(\frac{\bar{Z}}{\bar{z}^{*}_{st}}\right) \quad (1.8)$$

where

$\bar{x}^{*}_{st} = \sum_{h=1}^{L} W_h \bar{x}^{*}_h$ and $\bar{z}^{*}_{st} = \sum_{h=1}^{L} W_h \bar{z}^{*}_h$.

Bahl and Tuteja (1991) proposed ratio and product type exponential estimators for estimating the population mean $\bar{Y}$ in simple random sampling as

$$\hat{\bar{Y}}_{\text{Re}} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad (1.9)$$

and

$$\hat{\bar{Y}}_{Pe} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right). \quad (1.10)$$

Singh et al. (2008) modified Bahl and Tuteja (1991) estimators in stratified random sampling and suggested the following estimators

$$\hat{\bar{Y}}^{st}_{\text{Re}} = \bar{y}_{st}\exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right) \quad (1.11)$$

and

$$\hat{\bar{Y}}^{st}_{Pe} = \bar{y}_{st}\exp\left(\frac{\bar{x}_{st}-\bar{X}}{\bar{x}_{st}+\bar{X}}\right). \quad (1.12)$$

2. Suggested Estimators

Motivated by Srivenkataramana(1980) and Bahl and Tuteja (1991), using the transformation $x^{*}_{hi} = \frac{\bar{X}_h N_h - x_{hi} n_h}{N_h - n_h}$ in the h^{th} stratum, dual to Singh et al. (2008) estimators are suggested as

$$\hat{\bar{Y}}^{*ST}_{\mathrm{Re}} = \bar{y}_{st} \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{x}_h - \bar{X}_h)}{\sum_{h=1}^{L} W_h (\bar{x}_h + \bar{X}_h)} \right) \tag{2.1}$$

where $\bar{x}^*_h = \dfrac{\bar{X}_h N_h - \bar{x}_h n_h}{N_h - n_h}$.

Similarly, using the transformation $z^*_{hi} = \dfrac{\bar{Z}_h N_h - z_{hi} n_h}{N_h - n_h}$ dual to product type estimator is suggested as

$$\hat{\bar{Y}}^{*ST}_{Pe} = \bar{y}_{st} \exp\left(\frac{\sum_{L=1}^{h} W_h (\bar{Z}_h - \bar{z}^*_h)}{\sum_{L=1}^{h} W_h (\bar{Z}_h + \bar{z}^*_h)} \right) \tag{2.2}$$

where $\bar{z}^* = \dfrac{\bar{Z}_h N_h - \bar{z}_h n_h}{N_h - n_h}$.

To obtain the bias and mean squared error of the suggested ratio and product type exponential estimators, we write

$\bar{y}_h = \bar{Y}(1+e_{oh})$, $\bar{x}_h = \bar{X}(1+e_{1h})$ and $\bar{z}_h = \bar{Z}(1+e_{2h})$ such that $e_{oh} = e_{1h} = e_{2h} = 0$ and

$$E(e^2_{oh}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) C^2_{yh} = \gamma_h C^2_{yh},$$

$$E(e^2_{1h}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) C^2_{xh} = \gamma_h C^2_{xh},$$

$$E(e^2_{2h}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) C^2_{zh} = \gamma_h C^2_{zh},$$

$$E(e_{oh} e_{1h}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \rho_{yxh} C_{yh} C_{xh},$$

$$E(e_{oh} e_{2h}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \rho_{yzh} C_{zh} C_{zh}$$

and

$$E(e_{1h} e_{2h}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \rho_{xzh} C_{xh} C_{zh}.$$

Expressing (2.1) in term of e_{ih} we have

$$\hat{\bar{Y}}_{\mathrm{Re}}^{st} = \sum W_h \bar{Y}(1+e_{oh})\exp\left(\frac{-\sum W_h g_h \bar{X}_h e_{1h}}{2\sum W_h \bar{X}_h - \sum W_h g_h \bar{X}_h e_{1h})}\right)$$

$$\hat{\bar{Y}}_{\mathrm{Re}}^{st} = \bar{Y}(1+e_0)\exp\left[-\left(\frac{e_1}{2}\right)\left(1-\frac{e_1}{2}\right)^{-1}\right]$$

where $e_0 = \dfrac{\sum_{h=1}^{L} W_h \bar{Y}_h e_{1h}}{\bar{Y}}$, $e_1 = \dfrac{\sum_{h=1}^{L} W_h g_h \bar{X}_h e_{1h}}{\bar{X}}$ and $e_2 = \dfrac{\sum_{h=1}^{L} W_h g_h \bar{X}_h e_{2h}}{\bar{Z}}$

such that $E(e_o) = E(e_1) = E(e_2) = 0$ and

$$E(e_0^2) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \ , \ E(e_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 r_h g_h^2 S_{xh}^2 \ ,$$

$$E(e_2^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 r_h g_h^2 S_{zh}^2 \ , \ E(e_0 e_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} w_h^2 \gamma_h g_h S_{yxh} \ ,$$

$$E(e_0 e_2) = \frac{1}{\bar{Y}\,\bar{Z}}\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{yzh} \ \text{and} \ E(e_1 e_2) = \frac{1}{\bar{Y}\,\bar{Z}}\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{xzh} \ .$$

Finally, the bias and mean squared error of the suggested ratio type exponential estimator $\hat{\bar{Y}}_{\mathrm{Re}}^{*ST}$ are obtained as

$$B(\hat{\bar{Y}}_{\mathrm{Re}}^{*ST}) = -\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h^2 \gamma_h g_h \left(S_{yxh} + \frac{3}{4} R g_h S_{xh}^2\right) \tag{2.3}$$

$$MSE(\hat{\bar{Y}}_{\mathrm{Re}}^{*ST}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + \frac{R^2 g_h^2 S_{xh}^2}{4} - R g_h S_{yxh}\right) \tag{2.4}$$

similarly, the bias and mean squared error of the suggested product type exponential estimator $\hat{\bar{Y}}_{Pe}^{*ST}$ are obtained as

$$B(\hat{\bar{Y}}_{Pe}^{*ST}) = \frac{1}{\bar{X}}\sum_{h=1}^{L} W_h^2 \gamma_h \left(g_h S_{yxh} + \frac{5}{8} R g_h^2 S_{xh}^2\right) \tag{2.5}$$

$$MSE(\hat{\bar{Y}}_{Pe}^{*ST}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + \frac{R^2 g_h^2 S_{xh}^2}{4} + R g_h S_{yxh}\right). \tag{2.6}$$

where

$$S_{yh}^2 = \frac{1}{N_h - 1}\sum_{h=1}^{L}(y_{hi} - \bar{Y}_h)^2 \ ,$$

$$S_{xh}^2 = \frac{1}{N_h - 1}\sum_{h=1}^{L}(x_{hi} - \overline{X}_h)^2 ,$$

$$S_{zh}^2 = \frac{1}{N_h - 1}\sum_{h=1}^{L}(z_{hi} - \overline{Z}_h)^2 ,$$

$$S_{yxh} = \frac{1}{N_h - 1}\sum_{h=1}^{L}(y_{hi} - \overline{Y}_h)(x_{hi} - \overline{X}_h) ,$$

$$S_{yzh} = \frac{1}{N_h - 1}\sum_{h=1}^{L}(y_{hi} - \overline{Y}_h)(z_{hi} - \overline{Z}_h) ,$$

and

$$g_h = \frac{n_h}{N_h - n_h} .$$

3. Efficiency Compressions

Variance of simple mean $\overline{y}_{ST}$ in stratified random sampling, mean squared error of combined ratio estimator $\hat{\overline{Y}}_{Rc}$, combined product estimator $\hat{\overline{Y}}_{Pc}$ and Singh et al. (2008) estimators $\hat{\overline{Y}}_{\mathrm{Re}}^{ST}$ and $\hat{\overline{Y}}_{Pe}^{ST}$ are

$$V(\overline{y}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 , \tag{3.1}$$

$$MSE(\hat{\overline{Y}}_{Rc}) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}), \tag{3.2}$$

$$MSE(\hat{\overline{Y}}_{RC}^{*}) = \sum_{h=1}^{L} W_h^2 r_h (S_{yh}^2 + R^2 g_h^2 S_{xh}^2 - 2Rg_h S_{yxh}) , \tag{3.3}$$

$$MSE(\hat{\overline{Y}}_{Pc}) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R^2 S_{xh}^2 + 2RS_{yxh}) , \tag{3.4}$$

$$MSE(\hat{\overline{Y}}_{PC}^{*}) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R^2 g_h^2 S_{xh}^2 + 2Rg_h S_{yxh}) , \tag{3.5}$$

$$MSE(\hat{\overline{Y}}_{\mathrm{Re}}^{ST}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + \frac{R^2 S_{xh}^2}{4} - RS_{yxh} \right) , \tag{3.6}$$

$$MSE(\hat{\bar{Y}}_{Pe}^{ST}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + \frac{R^2 S_{xh}^2}{4} + RS_{yxh} \right). \tag{3.7}$$

From (2.4), (3.1), (3.2), (3.3) and (3.6), it is observed that the suggested ratio type exponential estimator $\hat{\bar{Y}}_{\mathrm{Re}}^{*ST}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$R < \frac{4\sum_{h=1}^{L} W_h^2 \gamma_h S_{yxh}}{\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{xh}^2}, \tag{3.8}$$

(ii) $\hat{\bar{Y}}_{Rc}$ if

$$R > \frac{\sum_{h=1}^{L} W_h^2 \gamma_h (2S_{yxh} - g_h)}{\sum_{h=1}^{L} W_h^2 \gamma_h S_{xh}^2 (1 - g_h^2)}, \tag{3.9}$$

(iii) $\hat{\bar{Y}}_{RC}^{*}$ if

$$R > \frac{4\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{yxh}}{3\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{xh}^2}, \tag{3.10}$$

(iv) $\hat{\bar{Y}}_{\mathrm{Re}}^{ST}$ if

$$R > \frac{\sum_{h=1}^{L} w_h^2 \gamma_h S_{yxh} (1 - g_h)}{\sum_{h=1}^{L} w_h^2 \gamma_h S_{xh}^2 (1 - g_h^2)}. \tag{3.11}$$

From (2.6), (3.1), (3.4), (3.5) and (3.7), it is observed that the suggested product type exponential estimator $\hat{\bar{Y}}_{Pe}^{*ST}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$R < -\frac{4\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{yxh}}{\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{xh}^2}, \tag{3.12}$$

(ii) $\hat{\bar{Y}}_{PC}$ if

$$R < \frac{4\sum_{h=1}^{L} W_h^2 \gamma_h S_{yxh}(2-g_h)}{\sum_{h=1}^{L} W_h^2 \gamma_h (g_h^2 - 4)} , \tag{3.13}$$

(iii) $\hat{\bar{Y}}_{PC}^{*}$ if

$$R > -\frac{4\sum_{h=1}^{L} W_h^2 \gamma_h g_h S_{yxh}}{3\sum_{h=1}^{L} W_h^2 \gamma_h g_h^2 S_{xh}} , \tag{3.14}$$

(iv) $\hat{\bar{Y}}_{Pe}^{ST}$ if

$$R < \frac{4\sum_{h=1}^{L} W_h^2 \gamma_h S_{yxh}(1-g_h)}{3\sum_{h=1}^{L} W_h^2 \gamma_h S_{xh}^2 (g_h^2 - 1)} . \tag{3.15}$$

4. Empirical Study: To judge the performance of the suggested estimators $\hat{\bar{Y}}_{\mathrm{Re}}^{*ST}$ and $\hat{\bar{Y}}_{Pe}^{*ST}$, we consider two natural population data sets. Description of the population are given below:

Population I [Source: Murthy (1967), p. 228]

N=10	N_1=5	N_2=5
	$n_1 = 2$	$n_2 = 2$
	$\bar{X}_1$=214.40	$\bar{X}_2$=333.80
	$\bar{Y}_1$=1925.80	$\bar{Y}_2$=3115.60
	$S_{x_1}^2$ =5605.84	$S_{x_2}^2$ =4401.76
	$S_{y_1}^2$ =379360.16	$S_{y_2}^2$ =115860.24
	S_{yx_1} =39360.69	S_{yx_2} =22356.52

Population II (Source : Japan Meteorological Society)
y : Rainy days, x: Total annual sunshine hours

N=20 n=8	N_1=10	N_2=10
	n_1=4	n_2=4
	$\overline{X}_1$=1630	$\overline{X}_2$=2036
	$\overline{Y}_1$=149.70	$\overline{Y}_2$=102.60
	C_{y_1}=0.09	C_{y_2}=0.122
	C_{x_1}=0.0627	C_{x_2}=0.05
	S_{x_1}=102.17	S_{x_2}=103.26
	S_{y_1}=13.47	S_{y_2}=12.61
	S_{yx_1}=-1073.00	S_{yx_2}=-655.30

Popu-lations	Estimators								
	$\overline{y}_{st}$	$\hat{\overline{Y}}_{Rc}$	$\hat{\overline{Y}}^*_{RC}$	$\hat{\overline{Y}}^{ST}_{\text{Re}}$	$\hat{\overline{Y}}^{*ST}_{\text{Re}}$	$\hat{\overline{Y}}_{PC}$	$\hat{\overline{Y}}^*_{PC}$	$\hat{\overline{Y}}^{ST}_{Pe}$	$\hat{\overline{Y}}^{*ST}_{Pe}$
I	100.00	239.88	71.07	355.66	412.88	*	*	*	*
II	100.00	*	*	*	*	168.15	150.48	178.13	183.06

* Not applicable

Table 4.1: Percent relative efficiencies of $\overline{y}_{st}$, $\hat{\overline{Y}}_{RC}$, $\hat{\overline{Y}}^*_{RC}$, $\hat{\overline{Y}}^{ST}_{\text{Re}}$, $\hat{\overline{Y}}^{*ST}_{\text{Re}}$, $\hat{\overline{Y}}_{Pc}$, $\hat{\overline{Y}}^*_{PC}$, $\hat{\overline{Y}}^{ST}_{Pe}$ and $\hat{\overline{Y}}^{*ST}_{Pe}$ with respect to $\overline{y}_{st}$

Conclusion

Section 3 provides the conditions under which the suggested estimators have less mean squared error in comparison to other considered estimators. Table 4.1 exhibits that the suggested estimator $\hat{\overline{Y}}^{*ST}_{\text{Re}}$ has the highest percent relative efficiency in comparison to usual unbiased estimator $\overline{y}_{st}$, combined ratio estimator $\hat{\overline{Y}}_{Rc}$, dual to combined ratio estimator $\hat{\overline{Y}}^*_{RC}$ given by Kushwaha et al. (1990) and Singh et al. (2008) ratio-type estimator $\hat{\overline{Y}}^{ST}_{\text{Re}}$ for population I in which the study variate and auxiliary variate are positively correlated. For the population II in which the study variate and auxiliary variate are negatively correlated, the suggested estimator $\hat{\overline{Y}}^{*ST}_{Pe}$ has the highest percent relative efficiency in comparison to unbiased estimator $\overline{y}_{ST}$, combined product estimator $\hat{\bar{Y}}_{PC}$, dual to combined product estimator $\hat{\bar{Y}}_{PC}^{*}$ and Singh et al. (2008) product type estimator $\hat{\bar{Y}}_{Pe}^{ST}$.

Therefore, it can be concluded that the suggested estimators are more efficient than other considered estimators if conditions obtained in section 3 are satisfied and the suggested estimators are recommended for use in practice for estimating the finite population mean.

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