Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 6, Issue 2 (2013): 81-90

A STUDY OF LINEAR COMBINATION BASED FACTOR-TYPE ESTIMATOR FOR ESTIMATING POPULATION MEAN IN SAMPLE SURVEYS

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(Received April 26, 2013)

Abstract

The objective of this paper is to study the linear combination of factor-type (F-T) estimator to estimate population mean and its properties like bias, mean squared error (m.s.e.) etc. along with numerical study over different populations. The expressions of bias and mean squared error (m.s.e.) of the estimator are derived in the form of population parameters by using the concept of large sample approximations. The combination of F-T estimator is compared with some existing estimators and found better over existing estimators in case of negative correlation between study and auxiliary variables or equal efficient and tested by empirical study performed over several data sets.

Key Words: Linear Combination, Factor Type Estimator, Bias, Mean Squared Error.

1. Introduction:

The problem of estimation of population parameters is a burning issue in sample surveys and a number of methods have been advocated to improve the efficiency of estimators. In sample surveys, auxiliary information is used to select the units in the sample and for estimation of the population parameter, to improve the efficiency of the estimators. The ratio, product and regression techniques are used without any cause in sample surveys.

Cochran (2005) advocated on the use of additional information at estimation stage and discussed on ratio type estimator as well while coefficient of correlation between study variable and auxiliary variable is highly positive. Murthy (1964) presented product type estimator to estimate population mean while the study variable and auxiliary variable negatively correlated, Sisodia and Dwivedi (1981) utilized coefficient of variation of auxiliary variate. A dual to ratio estimator proposed by Srivenkataramana (1980). Singh and Tailor (2005) and Tailor and Sharma (2009) worked on ratio-cum-product estimators. Some other valuable contributions are Steel and Torrie (1960), Singh and Kumar (2011), Singh et al. (2012), Sanaullah et al. (2012) etc.

Singh and Shukla (1987) have discussed a family of factor-type (F-T) ratio estimator for estimating population mean. In continuation, one more contribution by Singh and Shukla (1993) is an efficient-factor–type (F-T) estimator for estimating the population mean. Singh and Agnihotri (2008) studied on a general class of estimators using auxiliary information in sample surveys and in the same they considered a linear combination of two estimators and proved the suggested one is better than other existing estimators. Sharma and Tailor (2010) considered a linear combination of ratio and dual to ratio estimator and proved that the ratio-cum-dual to ratio estimator of finite population mean in simple random sampling is better than some other existing estimators. Deriving motivation from all these evidences, we are studying on a linear combination based factor-type (F-T) estimator to estimate population mean.

Let a simple random sample of size n is drawn from a finite population $U = \{1,2,3,..., N\}$ of size N. Let (y_i, x_i) be the pairs of observations, where y_i is the value of variable Y under consideration and x_i is the value of auxiliary variable X of i^{th} individual.

The motivation is derived from Singh and Shukla (1987) and they have discussed a family of factor type ratio estimator for estimating population mean. The factor type estimator is given by Singh and Shukla (1987) as

$$\bar{y}_{FT} = \bar{y}\frac{(A+C)\bar{X} + fB\bar{x}}{(A+fB)\bar{X} + C\bar{x}} \qquad ...(1.1)$$

where, $A = (k-1)(k-2)$; $B = (k-1)(k-4)$; $C = (k-2)(k-3)(k-4)$; $0 \le k \le \infty$; $f = n/N$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ being the sample mean of study variable and auxiliary variable respectively.

For the study and comparison of any estimator we usually obtain the expression of the bias and mean squared error (m.s.e.) of estimator in the form of parameters and for this purpose the concept of large sample approximations used. Under this, for a large sample, consider $\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$, where, $|e_0| < 1$ and $|e_1| < 1$ are error terms. Hence, we have $E(e_0) = E(e_1) = 0$, $E(e_0^2) = \left(\frac{1}{n} - \frac{1}{N}\right)C_y^2$, $E(e_1^2) = \left(\frac{1}{n} - \frac{1}{N}\right)C_x^2$ and $E(e_0e_1) = \left(\frac{1}{n} - \frac{1}{N}\right)\rho C_y C_x$.

By using the concept of large sample approximations the expressions of bias and mean squared error (m.s.e.) of the factor type (F-T) estimator $\bar{y}_{FT}$ in the form of population parameters are given by:

$$B(\bar{y}_{FT}) = \bar{Y}(\theta_2 - \theta_1)\left(\frac{1-f}{N}\right)\left(\theta_2 C_X^2 - \rho C_Y C_X\right) \qquad ...(1.2)$$

$$M(\bar{y}_{FT}) = \bar{Y}^2\left(\frac{1-f}{n}\right)\left[C_Y^2 + \theta^2 C_X^2 + 2\theta\rho C_Y C_X\right] \qquad ...(1.3)$$

respectively and the equation of minimum mean squared error of $\bar{y}_{FT}$ at $\theta = \theta_1 - \theta_2 = \rho\frac{C_Y}{C_X} = -V$ (say) is given by:

$$M(\bar{y}_{FT})_{min} = \left(\frac{1-f}{n}\right)(1-\rho^2)S_Y^2 \qquad ...(1.4)$$

Note that on simplifying the relation $\theta = -V$, we have the cubic equation in the form of k which is given by:

$$(V-1)k^3 + (fV + f - 8V + 9)k^2 - (5fV + 5f - 23V + 26)k + (4fV + 4f - 22V + 24) = 0 \qquad ...(1.5)$$

By solving this expression one can get at most three values of k like k_1, k_2 and k_3 for which m.s.e. is optimal.

Remark 1: Define,

$\theta = \theta_1 - \theta_2$, $\theta_1 = \dfrac{fB}{A + fB + C}$, $\theta_2 = \dfrac{C}{A + fB + C}$, $\theta_3 = \dfrac{A + C}{A + fB + C}$, $\theta_4 = \dfrac{A + fB}{A + fB + C}$ $C_y = \dfrac{S_Y}{Y}$,

$C_x = \dfrac{S_X}{X}$, $V = \rho \dfrac{C_Y}{C_X}$, $f = \dfrac{n}{N}$, $S_Y^{\,2} = \dfrac{1}{N-1}\sum\left(y_i - \overline{Y}\right)^2$, $S_Y^{\,2} = \dfrac{1}{N-1}\sum\left(x_I - \overline{X}\right)^2$

and ρ_{XY} = correlation coefficient between X and Y.

Also, $\theta_1 + \theta_3 = 1 = \theta_2 + \theta_4$, $\theta_1 - \theta_2 = -(\theta_3 - \theta_4)$,

2. The Linear Combination Based F-T Estimator

Under the same situation as discussed before we consider linear combination of factor-type estimator as follows:

$$\overline{y}_{FTRP} = f\phi(k) + (1 - f)\psi(k) \qquad \text{...(2.1)}$$

where,

$$\phi(k) = \overline{y}\left[\frac{(A + C)\overline{X} + fB\,\overline{x}}{(A + fB)\overline{X} + C\,\overline{x}}\right];\ \psi(k) = \overline{y}\left[\frac{(A + C)\overline{x} + fB\,\overline{X}}{(A + fB)\overline{x} + C\,\overline{X}}\right]$$

$$A = (k-1)(k-2);\ B = (k-1)(k-4);\ C = (k-2)(k-3)(k-4);\ 0 \le k \le \infty$$

2.1 Some Special Cases

The estimator $\overline{y}_{FTRP}$ provides linear combination of some existing estimators on some special values of k as shown in table 1.

Table 1: The Estimator $\overline{y}_{FTRP}$ in Special Cases

Value of *k*	Estimator	Remark
$k = 1$	$\left[\overline{y}_{FTRP}\right]_{k=1} = f\overline{y}\dfrac{\overline{X}}{\overline{x}} + (1-f)\overline{y}\dfrac{\overline{x}}{\overline{X}}$	Linear combination of Ratio-Product type estimator.
$k = 2$	$\left[\overline{y}_{FTRP}\right]_{k=2} = f\overline{y}\dfrac{\overline{x}}{\overline{X}} + (1-f)\overline{y}\dfrac{\overline{X}}{\overline{x}}$	Linear combination of Product-Ratio type estimator.
$k = 3$	$\left[\overline{y}_{FTRP}\right]_{k=3} = f\overline{y}\dfrac{\overline{X} - f\overline{x}}{(1-f)\overline{X}} + \overline{y}\dfrac{\overline{x} - f\overline{X}}{\overline{X}}$	Linear combination of Dual To Ratio-Product type estimator.
$k = 4$	$\left[\overline{y}_{FTRP}\right]_{k=4} = \overline{y}$	Unbiased estimator.

It is clear that from the table 1, $\overline{y}_{FTRP}$ provides the linear combination of ratio and product type estimator at k = 1 and at k = 2. Also, at k = 3 it provides the linear combination of dual to ratio estimator and at k = 4 suggested one is unbiased estimator.

3. Properties of the Estimator $\bar{y}_{FTRP}$

In this section, we shall focus on the bias and mean squared error (m.s.e.) of the estimator $\bar{y}_{FTRP}$ along with its minimum m.s.e. at its optimum value of *k*. The estimator $\bar{y}_{FTRP}$ could be written in terms of e_0 and e_1 as

$$\bar{Y}\left[1+e_0+(\theta_1-\theta_2)\left\{(2f-1)e_1+(\theta_4-f)e_1^2+(2f-1)e_0e_1\right\}\right] \qquad \text{...(3.1)}$$

and the estimator $\bar{y}_{FTRP}$ is found to be biased and it is in the form of parameters is:

$$B(\bar{y}_{FTRP})=\bar{Y}(\theta_1-\theta_2)\left(\frac{1}{n}-\frac{1}{N}\right)\left\{(\theta_4-f)C_X^2+(2f-1)\rho C_Y C_X\right\} \qquad \text{...(3.2)}$$

Also, the m.s.e. of $\bar{y}_{FTRP}$ is:

$$M(\bar{y}_{FTRP})=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2+(\theta_1-\theta_2)^2(2f-1)^2C_X^2+2(\theta_1-\theta_2)(2f-1)\rho C_Y C_X\right] \qquad \text{...(3.3)}$$

and minimum m.s.e. of $\bar{y}_{FTRP}$ at $\theta_1-\theta_2=-\dfrac{V}{2f-1}$ is

$$M(\bar{y}_{FTRP})_{min}=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2+V^2C_X^2-2V\rho C_X C_Y\right] \qquad \text{...(3.4)}$$

The symbols have their usual meaning and some specific symbols are already defined in Remark 1 and the proofs of above expressions are done in Appendix **A** as well.

Remark 2: For getting the optimum value of *k* by solving the relation $\theta=-V$, the cubic equation in the form of *k* is given by:

$$(V-1)k^3+(2f^2+20V+38)k^2-(10f+12V+61)k+(8f^2-26V+22)=0$$

Remark 3: The relation $\theta_1-\theta_2=-\dfrac{V}{2f-1}$ could be written as $f=\dfrac{1}{2}\left(1-\dfrac{V}{\theta_1-\theta_2}\right)$ ...(3.5)

Obviously, for a fix value of *k* and *V* we can calculate optimum value of sampling fraction *f* and using this value one could get the optimum sample size for known population size *N* since $f=\dfrac{n}{N}$. For example, at $k=1$, $\theta_1=0$, $\theta_2=1$ and we have $f=\dfrac{1}{2}(1+V)\Rightarrow n=\dfrac{N(1+V)}{2}$.

4. Comparisons

(1) The variance of $\bar{y}$ in SRSWOR is given by

$$V(\bar{y})=\left(\frac{1}{n}-\frac{1}{N}\right)S_Y^2=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)C_Y^2$$

and the minimum mean squared error (m.s.e.) of $\bar{y}_{FTRP}$ is:

$$M(\bar{y}_{FTRP})_{min}=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^2+V^2C_X^2-2V\rho C_X C_Y\right]$$

Now, let

$$D_1=V(\bar{y})-M(\bar{y}_{FTRP})_{min}$$

$$=\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)C_Y^2-\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_Y^2+V^2C_X^2-2V\rho C_Y C_X\right)$$

$$= -\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[V^2 C_X{}^2 - 2V\rho C_Y C_X\right]$$

$\bar{y}_{FTRP}$ is better than $\bar{y}$ if $D_1 > 0$

$$\Rightarrow -\bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[V^2 C_X{}^2 - 2\rho V C_Y C_X\right] > 0 \qquad \Rightarrow \rho < 0$$

Obviously, if there is negative correlation between *X* and *Y* then $\bar{y}_{FTRP}$ is better than $\bar{y}$.

(2) The mean squared error of $\bar{y}_R$ is:

$$M\left(\bar{y}_R\right) = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_X{}^2 + C_Y{}^2 - 2\rho C_Y C_X\right)$$

Let $D_2 = M\left(\bar{y}_R\right) - M\left(\bar{y}_{FTRP}\right)_{min}$

$$= \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_X{}^2 + C_Y{}^2 - 2\rho C_Y C_X\right) - \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_Y{}^2 + V^2 C_X{}^2 - 2V\rho C_Y C_X\right)$$

$\bar{y}_{FTRP}$ is better than $\bar{y}_R$ if $D_2 > 0 \Rightarrow \rho C_Y > \dfrac{C_X(1+V)}{2} \Rightarrow \rho > \dfrac{C_X}{C_Y}$

(3) Let $D_3 = M\left(\bar{y}_P\right) - M\left(\bar{y}_{FTRP}\right)_{min}$

$$= \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_X{}^2 + C_Y{}^2 + 2\rho C_Y C_X\right) - \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_Y{}^2 + V^2 C_X{}^2 - 2V\rho C_Y C_X\right)$$

$\bar{y}_{FTRP}$ is better than $\bar{y}_P$ if $D_3 > 0 \Rightarrow \rho C_Y > \dfrac{C_X(1+V)}{2} \Rightarrow \rho > \dfrac{C_X}{C_Y}$

(4) The mean squared error of $\bar{y}_{ST}$ is:

$$M\left(\bar{y}_{ST}\right) = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left(C_X{}^2 + \left(\frac{n}{N-n}\right)^2 C_Y{}^2 - 2\left(\frac{n}{N-n}\right)\rho C_Y C_X\right)$$

Let $D_4 = M\left(\bar{y}_{ST}\right) - M\left(\bar{y}_{FTRP}\right)_{min}$

$$= \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[\left(C_X{}^2 + \left(\frac{n}{N-n}\right)^2 C_Y{}^2 - 2\left(\frac{n}{N-n}\right)\rho C_Y C_X\right) - \left(C_Y{}^2 + V^2 C_X{}^2 - 2V\rho C_Y C_X\right)\right]$$

$$= \left(1 - V^2\right)C_X{}^2 + C_Y{}^2\left[\left(\frac{n}{N-n}\right)^2 - 1\right] - 2\rho C_Y C_X\left[\left(\frac{n}{N-n}\right) - V\right]$$

$$= \left(1 - \rho^2\frac{C_Y{}^2}{C_X{}^2}\right)C_X{}^2 + C_Y{}^2\left[\left(\frac{n}{N-n}\right)^2 - 1\right] - 2\rho C_Y C_X\left[\left(\frac{n}{N-n}\right) - \rho\frac{C_Y}{C_X}\right]$$

$$= C_X{}^2 - \rho^2 C_Y{}^2 + C_Y{}^2\left[\left(\frac{n}{N-n}\right)^2 - 1\right] - 2\rho C_Y C_X\frac{n}{N-n} + 2\rho^2 C_Y{}^2$$

$$= C_Y{}^2\rho^2 - \left(\frac{2nC_Y C_X}{N-n}\right)\rho + \left[C_X{}^2 + C_Y{}^2\left\{\left(\frac{n}{N-n}\right)^2 - 1\right\}\right]$$

$\bar{y}_{FTRP}$ is better than $\bar{y}_{ST}$ if $D_4 > 0$

$$\Rightarrow C_Y{}^2\rho^2 - \left(\frac{2n}{N-n}C_Y C_X\right)\rho + \left[C_X{}^2 + C_Y{}^2\left\{\left(\frac{n}{N-n}\right)^2 - 1\right\}\right] > 0 \qquad \ldots(4.1)$$

which is a quadratic inequality in ρ and it will have real roots if

$$\left(\frac{2nC_Y C_X}{N-n}\right)^2 - 4C_Y{}^2\left[C_X{}^2 + C_Y{}^2\left\{\left(\frac{n}{N-n}\right)^2 - 1\right\}\right] > 0$$

$$\Rightarrow N\left(C_X^2 - C_Y^2\right)(2n-N) > 0 \quad \Rightarrow C_X > C_Y \qquad or \qquad n > \frac{N}{2}$$

and the roots of ρ could be obtained by considering the inequality (4.1) as equality using the relation as below

$$\Rightarrow \rho = \frac{nC_X \pm \sqrt{N\left(C_X^2 - C_Y^2\right)(2n-N)}}{(N-n)C_Y} \ .$$

(5) The optimum m.s.e. of factor-type estimator is $M(\bar{y}_{FT})_{min} = \left(\frac{1-f}{n}\right)(1-\rho^2)S_Y{}^2$

Again, let $D_5 = M(\bar{y}_{FT})_{min} - M(\bar{y}_{FTRP})_{min} = 0$ therefore, both the estimators are equal efficient.

5. Empirical Study

To examine the performance of the estimator $\bar{y}_{FTRP}$ in comparison to the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_{ST}$, $\bar{y}_{FT}$ and we have considered five data sets as A, B, C, D and E as given in the table-1:

Table-1: Different Population under Consideration

Parameters	**Population A**	**Population B**	**Population C**	**Population D**	**Population E**
Source	Kadilar and Cingi (2006)	Shukla and Thakur (2008)	Pandey and Dubey (1988)	Maddala (1977)	Steel and Torrie (1960)
f	0.188679	0.1	0.4	0.2	0.20
$\bar{Y}$	15.37	42.485	19.55	7.6375	0.6860
$\bar{X}$	243.76	18.515	18.8	75.4313	0.8077
C_Y	4.18	0.3321	0.1445	0.2278	0.700123
C_X	2.02	0.3763	0.1281	0.0986	0.7493
ρ	0.82	0.8652	-0.9199	-0.6823	-0.4996
V	1.696832	0.7635	-0.74657	-1.57634	-0.57637
N	106	200	20	30	30
n	20	20	8	6	6

Table: 2 Bias and M.S.E. of Different Estimators on Different Population

Populations	Estimators	k	Bias	M.S.E.	PRE
Population A	$\bar{y}$	---	0	167.4	100
	$\bar{y}_R$	---	-1.7728	73.84	44.099
	$\bar{y}_P$	---	4.3169	339.2	202.60
	$\bar{y}_{ST}$	---	-1.0039	166.3	99.339
	$\bar{y}_{FT}$	k_1=2.3725	-3.7401	55.23	32.987
		k_2^*	---	---	
		k_3^*	---	---	
	$\bar{y}_{FTRP}$	k_1=1.2598	-48.393	167.59	100.092
		k_2^*	---	---	
		k_3^*	---	---	
Population B	$\bar{y}$	---	0	8.9582	5.3500
	$\bar{y}_R$	---	0.0640	2.895	1.7290
	$\bar{y}_P$	---	0.2067	60.84	36.335
	$\bar{y}_{ST}$	---	-0.0229	9.076	5.4205
	$\bar{y}_{FT}$	k_1=1.5191	0.0123	2.252	1.3451
		k_2=9.0124	0.0030	2.252	1.3451
		k_3=2.4423	0.1606	2.252	1.3451
	$\bar{y}_{FTRP}$	k_1=0.0032	0.2512	8.970	5.3574
		k_2=1.3401	0.1312	8.963	5.3533
		k_3^*	---	---	
Population C	$\bar{y}$	---	0	0.598	0.3574
	$\bar{y}_R$	---	0.0490	2.045	1.2213
	$\bar{y}_P$	---	-0.0249	0.092	0.0553
	$\bar{y}_{ST}$	---	0.0166	0.830	0.4958
	$\bar{y}_{FT}$	k_1=1.9551	-0.0363	0.132	0.0790
		k_2^*	---	---	
		k_3^*	---	---	
	$\bar{y}_{FTRP}$	k_1=10.9249	-1.6292E-05	0.5985	0.3574
		k_2^*	---	---	
		k_3^*	---	---	
Population D	$\bar{y}$	---	0	0.4035	0.2410
	$\bar{y}_R$	---	0.0255	0.7175	0.4285
	$\bar{y}_P$	---	-0.0156	0.2408	0.1438
	$\bar{y}_{ST}$	---	0.0039	0.4159	0.2484
	$\bar{y}_{FT}$	k_1=1.9700	-0.0115	0.2761	0.1649
		k_2^*	---	---	
		k_3^*	---	---	
	$\bar{y}_{FTRP}$	k_1=12.3769	0.0073	0.4034	0.2409
		k_2^*	---	---	

Populations	Estimators	k	Bias	M.S.E.	PRE
		k_3^*	---		
Population E	$\bar{y}$	---	0	0.0307	0.0183
	$\bar{y}_R$	---	0.0753	0.1014	0.0605
	$\bar{y}_P$	---	-0.0239	0.0330	0.0197
	$\bar{y}_{ST}$	---	0.0059	0.1640	0.0979
	$\bar{y}_{FT}$	k_1=1.85167	-2.53894E-07	0.0307	0.0183
		k_2^*	---	---	
		k_3^*	---	---	
	$\bar{y}_{FTRP}$	k_1=1.0001	-0.0041	0.0426	0.0254
		k_2=1.00011	-0.0041	0.0426	0.0254
		k_3^*	---	---	

5. Discussion and Conclusions

The Linear combination of Factor-Type estimator may be considered as an estimator of the population mean and properties like bias, m.s.e., etc. of the estimators could be derived in terms of population parameters under the concept of large sample approximations and a comparative study with existing estimators can be established. For the same we considered the linear combination of Factor-Type estimator and derive its bias and mean squared error. It is found that the proposed estimator is better over existing estimator if there is negative correlation between study and ancillary variable. i.e. suggested estimator have minimum mean squared error (m.s.e.) in case the study and auxiliary variable are negatively correlated or equal efficient if not. Factor-type and Factor-type ratio product estimator both are equal efficient in term of parameters at optimum values of constant. Empirical study has been done over five populations and the bias and m.s.e. of suggested and existing estimators calculated. Percentage relative efficiency of the estimators has been calculated as well. We observed that where positive correlation between the variables then the suggested estimator is approximate equal efficient and for negative correlation it is better over existing. Also, there is the choice of *k* for minimum bias and for specified value of *k* we can obtain optimum choice of sample size.

Acknowledgement

Authors are thankful to the Editorial Board of JRSS and referees for recommending the manuscript for publication and their valuable suggestions.

References

1. Cochran, W. G. (1977). Sampling Techniques, Third U. S. Edition, Wiley Eastern Limited. a. 325.
2. Das, A.K. (1988). Contribution to the theory of sampling strategies based on auxiliary information. Ph. D. Thesis, BCKV, West Bengal, India.
3. Kadilar C. and Cingi, H. (2006). Improvement in estimating the population mean in simple random sampling, Elsevier, 19, p. 75–79.
4. Maddala, G.S.(1977). Econometrics. New York, McGraw-Hill.
5. Murthy, M.N. (1967). Sampling theory and methods, Statistical Publishing Society, Calcutta,

6. Murty, M. N. (1964). Product method of estimation, Sankhya, A, 26, p. 294-307.
7. Pandey, B.N. and Dubey, V. (1988). Modified product estimator using coefficient of variation of auxiliary variable, Assam Stat. Rev., 2, p. 64-66.
8. Sanaullah, A., Khan, H., Ali, H. A., Singh, R. (2012). Improved exponential ratio-type estimators in survey sampling, Jour. Reliab. and Stat. Stud. 5(2), p. 119-132.
9. Sharma, B and Tailor, R. (2010). A New Ratio-Cum-Dual to Ratio Estimator of Finite Population Mean in Simple Random Sampling, Global Journal of Science Frontier Research, Vol. 10 Issue 1 (Ver 1.0), p. 27-31.
10. Shukla, D. and Thakur, N.S. (2008). Estimation of Mean with Imputation of Missing Data Using Factor Type Estimator, Statistics in Transition, Vol. 9, No. 1, p. 33-48.
11. Shukla, D., Pathak, S. and Thakur, N.S. (2012). Estimation of Population Mean Using Two Auxiliary Sources in Sample Surveys, Statistics in Transition-new series, Vol. 13, No. 1, p. 21—36.
12. Singh, H. P. and Agnihotri, N. (2008). A General Class of Estimating Population Mean Using Auxiliary Information in Sample Surveys, Statistics in Transition, 9, 1, p. 71-87.
13. Singh, H. P. and Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean, Statistics in Transition, 6 (4), p. 555-560.
14. Singh, H. P. and Tailor, R. (2005). Estimation of finite population mean using known correlation coefficient between auxiliary characters, Statistica, Anno LXV, 4, p. 407-418.
15. Singh, R. and Kumar, M. (2011). A note on transformations on auxiliary variable in survey sampling, Mod. Assis. Stat. Appl., 6.1, 17-19.doi 10.3233/MAS-2011-0154
16. Singh, R., Malik, S., Chaudhary, M.K., Verma, H. and Adewara, A.A. (2012). A general family of ratio type- estimators in systematic sampling, Jour. Reliab. and Stat. Stud., 5(1), p. 73-82.
17. Singh, V. K. and Shukla, D. (1987). One parameter family of factor type ratio estimator, Metron, 45, 1-2, p. 273-283.
18. Singh, V. K. and Shukla, D. (1993). An efficient one parameter family of factor - type estimator in sample survey, Metron, 51, 1-2, p. 139-159.
19. Sisodia, B.V.S. and Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable, Jour. Ind. Soc. Agri. Stat., 33(1), p. 13-18.
20. Srivenkataramana, T. (1980). A dual of ratio estimator in sample surveys, Biometrika, 67, 1, p. 199-204.
21. Steel, R.G.D and Torrie, J. H. (1960). Principles and procedures of statistics, McGraw hill book.
22. Sukhatme, P. V. and Sukhatme, B. V. (1970). Sampling theory of surveys with applications, Iowa State University Press, Ames, U. S. A.
23. Tailor, R. and Sharma, B. K. (2009). A Modified Ratio-Cum-Product Estimator of Finite Population Mean Using Known Coefficient of Variation and Coefficient of Kurtosis. Statistics in Transition-new series, Jul-09, Vol. 10, No. 1, p. 15-24.

APPENDIX A

Theorem 1: The form of the estimator is given as

$$\bar{y}_{FTRP} = f\phi(k) + (1-f)\psi(k)$$

$$\bar{y}_{FTRP} = f\bar{y}\left[\frac{(A+C)\bar{X} + fB\,\bar{x}}{(A+fB)\bar{X} + C\,\bar{x}}\right] + (1-f)\bar{y}\left[\frac{(A+C)\bar{x} + fB\,\bar{X}}{(A+fB)\bar{x} + C\,\bar{X}}\right]$$

The large sample approximation form of the above estimator is

$$\bar{Y}(1+e_0)\left[f(1+\theta_1 e_1)(1+\theta_2 e_1)^{-1} + (1-f)(1+\theta_3 e_1)(1+\theta_4 e_1)^{-1}\right]$$

On solving it we get,

$$\bar{Y}\left[1 + e_0 + (\theta_1-\theta_2)\left\{(2f-1)e_1 + (\theta_4 - f)e_1^{\,2}\right\} + e_0 + (\theta_1-\theta_2)\left\{(2f-1)e_0 e_1 + (\theta_4 - f)e_0 e_1^{\,2}\right\}\right]$$

ignoring higher order terms we get,

$$\bar{y}_{FTRP} = \bar{Y}\left[1 + e_0 + (\theta_1-\theta_2)\left\{(2f-1)e_1 + (\theta_4 - f)e_1^{\,2} + (2f-1)e_0 e_1\right\}\right]$$

Theorem 2: The estimator $\bar{y}_{FTRP}$ is biased and it is given by:

$$B(\bar{y}_{FTRP}) = \bar{Y}(\theta_1-\theta_2)\left(\frac{1}{n}-\frac{1}{N}\right)\left\{(\theta_4 - f)C_X^{\,2} + (2f-1)\rho C_Y C_X\right\}$$

Proof: Since we know that

$$B(\bar{y}_{FTRP}) = E(\bar{y}_{FTRP} - \bar{Y})$$

On putting the values from equation (3.1) we get

$$B(\bar{y}_{FTRP}) = \bar{Y}(\theta_1-\theta_2)\left(\frac{1}{n}-\frac{1}{N}\right)\left\{(\theta_4 - f)C_X^{\,2} + (2f-1)\rho C_Y C_X\right\}$$

Theorem 3: The mean squared error of $\bar{y}_{FTRP}$ is given by

$$M(\bar{y}_{FTRP}) = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^{\,2} + (\theta_1-\theta_2)^2(2f-1)^2 C_X^{\,2} + 2(\theta_1-\theta_2)(2f-1)\rho C_Y C_X\right]$$

Proof: Since we know that

$$M(\bar{y}_{FTRP}) = E(\bar{y}_{FTRP} - \bar{Y})^2$$

Large sample approximation form is given by,

$$M(\bar{y}_{FTRP}) = \bar{Y}^2 E\left[e_0 + (\theta_1-\theta_2)(2f-1)e_1\right]^2$$

By simplifying the above expression and using the concept of large sample approximations we have

$$M(\bar{y}_{FTRP}) = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^{\,2} + (\theta_1-\theta_2)^2(2f-1)^2 C_X^{\,2} + 2(\theta_1-\theta_2)(2f-1)\rho C_Y C_X\right]$$

Theorem 4: The minimum M.S.E. of $\bar{y}_{FTRP}$ at $\theta_1 - \theta_2 = -\dfrac{V}{2f-1}$ is

$$M(\bar{y}_{FTRP})_{min} = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^{\,2} + V^2 C_X^{\,2} - 2V\rho C_X C_Y\right]$$

Proof: Diff. (3.3) w. r. to $P = \theta_1 - \theta_2$ (say) and equate to zero.

$$\frac{d}{dP}M(\bar{y}_{FTRP}) = \bar{Y}^2\left[2(\theta_1-\theta_2)(2f-1)^2\left(\frac{1}{n}-\frac{1}{N}\right)C_X^{\,2} + 2\left(\frac{1}{n}-\frac{1}{N}\right)(2f-1)\rho C_Y C_X\right] = 0$$

$$\Rightarrow (\theta_1-\theta_2)(2f-1)C_X + \rho C_Y = 0$$

$$\Rightarrow (\theta_1-\theta_2) = -\rho\frac{C_Y}{(2f-1)C_X} = -\frac{V}{(2f-1)}$$

Therefore the minimum m.s.e. for $\bar{y}_{FTRP}$ is given by

$$M(\bar{y}_{FTRP})_{min} = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{N}\right)\left[C_Y^{\,2} + V^2 C_X^{\,2} - 2V\rho C_X C_Y\right]$$