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EOQ Model for Time-Deteriorating Items Using Penalty cost

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Abstract

In inventory, the utility of the deteriorating items decreases with time. The degree of deterioration of product utility can be treated as penalty cost in the inventory replenishment system. In this paper, we present EOQ model for those perishable products, which do not deteriorate for some period of time and after that time they continuously deteriorate with time and loose their importance. This loss can be incurred as penalty cost to the wholesaler / retailer. The prime focus of our paper is to develop the EOQ model for time-deteriorating items using penalty cost with finite and infinite production rate. For simplicity, linear and exponential penalty cost functions have been considered as a measurement of the utility of the product. The theoretical expressions are obtained for optimum inventory level and cycle time. All the theoretical developments are numerically justified.

Key Words: Inventory, Time deteriorating items, Penalty cost.

1. Introduction

Considerable amount of research work has been devoted on decaying inventory system. In inventory, each item has either fixed lifetime or random lifetime. Fixed lifetime products have deterministic shelf life i.e. if a product remains unused up to its lifetime, it is considered to be out-dated and must be disposed off. These products are usually depleted following either First In First Out (FIFO) or Last In First Out (LIFO) issuing policy. Random lifetime products have not exact life. A typical example is fresh products whose time of spoilage is assumed to be a random variable.

Inventory models for fixed lifetime perishable products have been studied by Nandakumar and Morton [5] and Liu and Lina [4]. Perry [6] considered a perishable inventory system where the commodity's arrival and customer demand processes are stochastic and the stored items have a constant lifetime.

A stochastic dynamic programming model was developed by Jain and Silver [2] to determine the optimal ordering policy for a random lifetime perishable or potentially obsolete product. Liu and Chaung [3] developed a single item continuous review inventory models with Poisson demands, exponentially distributed lifetime and replenishment lead-times including all the possibilities of partial backlogging, complete backlogging and complete lost sales.

In both fixed and random lifetime perishability, the utility of an individual product or undecayed products is constant. But this is the fact that the utility of the product does not seem reasonably constant, since the selling price of many perishable products like fresh vegetables, fruits, milk products, bakery items etc. decreases with time. This loss is termed as a penalty cost to the wholesaler / retailer.

Fujiwara and Perera [1] have proposed an EOQ model for continuously deteriorating items using linear and exponential penalty costs. They have assumed that the utility of the perishable product concerned deteriorates continuously from the beginning of the replenishment cycle. The penalty cost at age zero is set to zero as no utility deterioration has occurred and penalty cost rises over time until it reaches the level of the original utility of the product, which can then be considered to have reached its lifetime.

But usually, in practice, most of the perishable products do not deteriorate for some period of time and after that time deteriorate continuously with time until the utility of the product reaches zero.

Products like fresh vegetables, fruits, dairy products, bakery items etc. do not deteriorate at the beginning of the period but they continuously deteriorate after some time. As a result of this, the selling price of such products decreases which can considered as a penalty cost for the wholesaler/retailer.

Taking into account this consideration, in the present paper, we have developed an EOQ model for infinite and finite production rate for time deteriorating items i.e. for those perishable products that do not deteriorate for some period of time and after that period they continuously deteriorate with time and become useless till they reach their lifetime. This loss can be incurred upon as a penalty cost to the wholesaler/ retailer and thus it has been incorporated in this proposed model.

In this context, we have considered two types of penalty cost function of age
(i) Linear
(ii) Exponential penalty cost functions, as a measurement of utility of the product.
A linear penalty cost function

$$P(t) = \pi (t - \mu), \quad t \geq \mu$$
$$= 0; \quad \text{otherwise}$$

which gives the cost of keeping one unit of product in stock until age t, where μ be that time period at which deterioration of product start and π is constant. There will be no penalty cost incurred upon the products up to time period (0, μ).

An exponential penalty cost function is taken as

$$P(t) = \alpha\left(e^{\beta(t-\mu)}-1\right), \quad t \geq \mu$$
$$= 0; \quad \text{otherwise}$$

which also gives the cost of keeping one unit of product in stock until age t, where μ be that time period at which deterioration of product start and α and β are constants.

EOQ model for infinite production rate has been developed in section (3) while for finite production rate is formulated in section (4). The sensitivity analysis has also been done in order to judge the effectiveness of the suggested models.

2. Notations And Assumptions

The proposed inventory model is developed under the following assumptions and notations.

2.1 Notations

Q	=	Number of items received at the beginning of the period.
D	=	Demand rate.
H	=	Inventory holding cost per unit per unit time.
A	=	Set-up cost per cycle.
μ	=	The time period at which deterioration of products start.
C(T)	=	Average total variable cost per unit time.
T	=	Length of replenishment cycle, which will not exceed product lifetime.
T*	=	Optimum value of T.
Q*	=	Optimum value of Q.

2.2 Assumptions

1. A single product is considered over a prescribed period of T unit of time.
2. The replenishment occurs instantaneously at an infinite rate.
3. The demand rate is constant say D units per unit of time.
4. No back order is permitted.
5. Delivery lead-time is zero.
6. The holding cost, ordering cost remains constant over time.

3. EOQ Model For Infinite Production Rate

3.1 Formulation and Solution

According to the assumptions and notations mentioned above, the, behavior of inventory system in one cycle may be depicted as in figure (1).

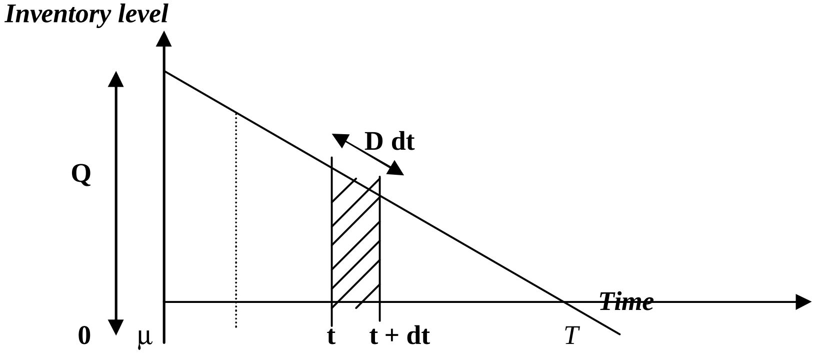

Fig.(1)

Let Q be the number of items received or the inventory level at the beginning of the period. From the figure (1), it is clear that inventory level decreases due to the constant demand say D units per unit of time. Up to the time interval (0, μ), there is no deterioration of the product but after time t = μ, the product continuously deteriorates so the penalty cost can be incurred upon for the time interval (μ, T).

The total variable cost per cycle time consists of the inventory holding cost, set up cost and penalty cost. Since the demand rate is D units per unit time,

∴ The total demand in one cycle of time-interval T is = DT
∴ The number of items received at the beginning of the period is Q = DT (1)

Let the age of the product delivered at time t is t. For the time interval (0, μ), no perishability occurs only demand is delivered during this period at the rate of D units per unit time. But for the time- interval (μ, T), deterioration of the product starts. As a result, penalty cost has been imposed.

Case I: When linear penalty cost function is used

A linear penalty cost function P(t) = π(t - μ), t $\geq$ μ which gives the cost of keeping one unit of product in stock until age t, where μ be that time period at which deterioration of product start and π is constant.

The cost due to the deterioration of the product delivered during the period (t, t + dt) is given by π(t - μ) D dt. Thus penalty cost due to the deterioration of the products delivered during the time interval (μ, T) is given by

$$\int_{\mu}^{T} \pi D\left(t-\mu\right)dt = \pi D\left[\frac{T^2}{2}-\mu T+\frac{\mu^2}{2}\right]$$

Now the cost of holding inventory for the period (0, T) is given by

$$H.\frac{1}{2}QT = \frac{1}{2}HDT^2 \qquad (\because Q = DT)$$

Therefore, the average total variable cost per unit time C (T) is given by

$$C\left(T\right) = \frac{1}{T}\left[A+\frac{\pi D\mu^2}{2}\right]+\left[\frac{HD}{2}+\frac{\pi D}{2}\right]T-\pi D\mu \qquad (2)$$

The optimal solution is obtained by differentiating C (T) with respect to T and equating it to zero. Then, the optimal cycle time T* is obtained and expressed as

$$T^* = \sqrt{\frac{2A+\pi D\mu^2}{\left(\pi+H\right)D}} \qquad (3)$$

The optimal economic order quantity Q* is obtained by putting the value of T* in eq. (1),

$$Q^* = \sqrt{\frac{D\left(2A+\pi D\mu^2\right)}{\left(\pi+H\right)}} \qquad (4)$$

From the above expressions (3) and (4), it is clear that if there is no perishability (i.e. $\pi=0$) then these two expressions become same as that of the non-perishable lot size model and if deterioration of the product starts at the beginning of the period (i.e. $\mu=0$) then (3) and (4) coincide with the expressions of the Fujiwara and Perera [1].

Case II: When exponential penalty cost function is used

An exponential penalty cost function P(t) = $\alpha\left(e^{\beta(t-\mu)}-1\right)$, $t\geq\mu$ which also gives the cost of keeping one unit of product in stock until age t, where μ be that time period at which deterioration of product start and α and β are constants.

The cost due to the deterioration of the product delivered during the period (t, t + dt) is given by $\alpha\left(e^{\beta(t-\mu)}-1\right)$ D dt.

The penalty cost due to the deterioration of the product delivered during the time interval (μ ,T) is given by

$$\int_{\mu}^{T} D\alpha\left(e^{\beta(t-\mu)}-1\right) dt = \frac{\alpha D}{\beta}\left[\left(e^{\beta(T-\mu)}-1\right)-\beta\left(T-\mu\right)\right]$$

Therefore, total variable cost per unit time is given by

$$C(T) = \frac{A}{T}+\frac{1}{2}HDT+\frac{\alpha D}{\beta T}\left[\left(e^{\beta(T-\mu)}-1\right)-\beta(T-\mu)\right]$$

By using second order approximation of the exponential term $e^{\beta(T-\mu)}$ in $C(T)$ we get,

$$C(T) = \frac{A}{T}+\frac{1}{2}HDT+\frac{1}{2}\alpha D\beta T+\frac{1}{2T}\alpha D\mu^2\beta-\alpha D\mu\beta$$

The optimal solution is obtained by differentiating C(T) with respect to T and equating it to zero. Then, the optimal cycle time T* is obtained and expressed as

$$T^* = \sqrt{\frac{2A+\alpha D\beta\mu^2}{D\left(H+\alpha\beta\right)}} \quad (5)$$

The optimal economic order quantity Q* is obtained by putting the value of T* in eq. (1),

$$Q^* = \sqrt{\frac{D\left(2A+\alpha D\beta\mu^2\right)}{\left(H+\alpha\beta\right)}} \quad (6)$$

From the expression (5) and (6), it is clear that if μ = 0 and $\alpha\beta=\pi$ then these two expressions are same as (3) and (4).

4. EOQ Model With Finite Production Rate

4.1 Formulation and Solution

In this model the assumption of infinite production rate is relaxed. The rate of production P units per period is finite. The behavior of an inventory system in one cycle may be depicted as in figure (2).

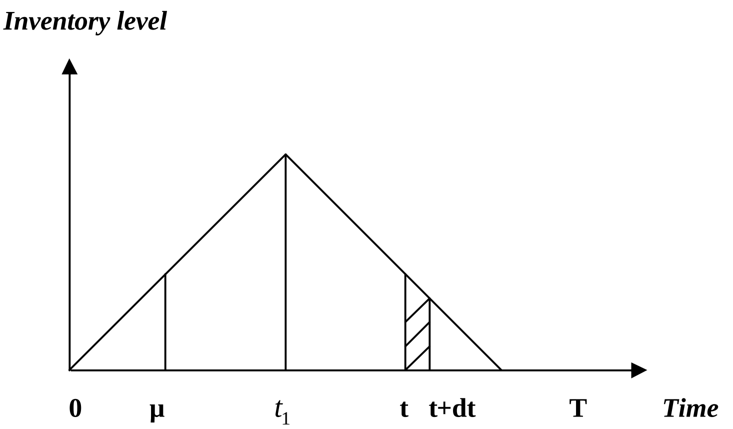

Fig.(2)

From the Fig.(2), it is clear that initially the stock is zero and the production starts with a finite rate P(> D) units per unit time while the demand is D unit per unit time. Thus, the inventory increases with a rate (P − D) units per unit time. Let the production continue for a period t_1.

∴ Inventory level at the and of the time t_1 is $q = (P - D)\ t_1$

Let P>D is the number of items produced per unit time. If 'Q' be the number of items produced per production run then production will continue for a time

$$t_1 = \frac{Q}{P}$$

The time of one complete cycle $T = \frac{Q}{D}$

After time t_1, production is completed. Then the inventory level at the moment when the production is completed is,

$$q = (P - D)\frac{Q}{P}$$

$$= Q\left(1 - \frac{D}{P}\right)$$

Let us assume that the products do not deteriorate at the beginning of the production cycle. But deteriorate after sometime of the production period. Let the perishability

occurs after time t = μ from the beginning of the cycle. The total variable cost per cycle time consists of the set up cost, inventory holding cost and perishability cost.
The holding cost per cycle is given by

$$\frac{1}{2}HqT = \frac{1}{2}HT^2D\left(1-\frac{D}{P}\right)$$

The total number of units delivered in time (t−μ), are $D(t-\mu)$ and their production time is $\frac{D(t-\mu)}{P}$, where $\mu < t \leq T$.
Therefore, the age of the product delivered at time‘t’ is given

$$\left[(t-\mu)-\frac{D(t-\mu)}{P}\right] = (t-\mu)\left(1-\frac{D}{P}\right)$$

Since the total number of units to be delivered during a period (t, t + dt) is D dt. If linear penalty cost function is used, then the cost due to deterioration of products delivered during the period $(t, t+dt)$ is given by

$$\pi(t-\mu)\left(1-\frac{D}{P}\right)D\ dt$$

Thus, the total cost due to the deterioration of products during one cycle is given by

$$\pi D\int_{\mu}^{T}(t-\mu)\left(1-\frac{D}{P}\right)dt = \pi D\left(1-\frac{D}{P}\right)\left(\frac{T^2+\mu^2}{2}-\mu T\right)$$

Therefore, the average total variable cost per period is given by

$$C(T) = \frac{A}{T}+\frac{1}{2}HTD\left(1-\frac{D}{P}\right)+\pi D\left(1-\frac{D}{P}\right)\left(\frac{T}{2}+\frac{\mu^2}{2T}-\mu\right)$$

The optimal solutions is obtained by differentiating C(T) and equate it to zero. Thus, the optimal cycle time T* and economic order quantity Q* are obtained and expressed

$$T^* = \sqrt{\frac{2A+\pi D\left(1-\frac{D}{P}\right)\mu^2}{D\left(1-\frac{D}{P}\right)(H+\pi)}} \qquad (7)$$

$$Q^* = T^*D$$

$$Q^* = \sqrt{\frac{D\left[2A+\pi D\left(1-\frac{D}{P}\right)\mu^2\right]}{(H+\pi)\left(1-\frac{D}{P}\right)}} \qquad (8)$$

If $P = \infty$ (i.e. production rate is infinite), then (7) and (8) coincide with the (3) and (4). Also if an exponential penalty cost function is used by using second order approximation of the exponential term $\exp\left(\beta\left(T-\mu\right)\left(1-\frac{D}{P}\right)\right)$ and by taking $\alpha\beta=\pi$, the same equations (7) and (8) are obtained.

5. Numerical Illustration

To illustrate the theoretical development, the following example has been considered.

P = 50 units per day, D = 25 units per day, H = Rs.0.01 per day

A = Rs. 100 per day, μ = 5 days, α = 10, β = 0.98

In case of EOQ model for infinite production rate

Case 1. when linear penalty cost function is used then,
Optimum cycle time T* = 5.24 days
and Optimum order quantity Q* = 131.00 units

Case 2. when exponential penalty cost function is used then,
Optimum cycle time T* = 5.08 days
and Optimum order quantity Q* = 126.96 units

In case of EOQ model for finite production rate

When linear penalty cost function is used then,
Optimum cycle time T* = 5.48 days
and Optimum order quantity Q* = 136.93 units

6. Sensitivity Analysis

The sensitivity analysis is performed for checking the effectiveness of the EOQ model for infinite production rate with respect to the parameter μ on optimum cycle time 'T*'and optimum order quantity 'Q*'. Similar analysis can also be done for finite production rate model. Table 1.and Table 2. depict the values of the optimum policies for different values of the parameter μ in case of linear and exponential penalty cost. Percentage changes of these values are shown with respect to the parameter μ = 5 in the data set taken for illustration.

μ	**Change(%) in μ**	**T***	**Q***
0	-100%	1.59 (-69%)	39.84 (-69%)
1	-80%	1.88 (-64%)	47.01 (-64%)
3	-40%	3.39 (-35%)	84.82 (-35%)
5	0%	5.24 (0%)	131.00 (0%)
7	40%	7.17 (37%)	179.20 (37%)
9	80%	9.13 (74%)	228.14 (74%)

Table 1: Effect of parameter μ on optimal policies in case of linear penalty cost

μ	Change(%) in μ	T*	Q*
0	-100%	0.90 (-82%)	22.58 (-82%)
1	-80%	1.34 (-74%)	33.68 (-74%)
3	-40%	3.13 (-38%)	78.29 (-38%)
5	0%	5.08 (0%)	126.96 (0%)
7	40%	7.05 (39%)	176.36 (39%)
9	80%	9.04 (78%)	226.02 (78%)

Table 2.: Effect of parameter μ on optimal policies in case of exponential penalty cost

The Table 1. shows the effect of parameter μ on optimum policies in case of linear penalty cost. If the value of the parameter μ is increased by 80%, the value of optimum cycle time increases by 74% and the optimum order quantity also increases by 74%. Further, if the parameter μ is decreased by 100%, the value of optimum cycle time decreases by 69% and the optimum order quantity also decreases by 69%.

When we examine for optimum policies in case of exponential penalty cost, we find that if the value of the parameter μ is increased by 80%, both the value of optimum cycle time and the optimum order quantity increases by 78%. Further, if the parameter μ is decreased by 100%, both the value of optimum cycle time and the optimum order quantity decreases by 82%.

Thus, we conclude that the values of the optimum cycle time and the optimum order quantity are not much sensitive to the change in the value of the parameter μ .

7. Conclusion

In this paper we have developed an EOQ model for time deteriorating items i.e. for those perishable products which do not deteriorate for some period of time and after that time they continuously deteriorate with time for both infinite and finite production rate. The penalty cost has also been incurred in the development of the model. A measure for the utility deterioration as a linear and an exponential penalty cost function are introduced into the model (as used by Fujiwara and Perera [1]). The sensitivity analysis has also been performed for finite production rate EOQ model to check the effectiveness of the proposed model.

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