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A CLASS OF ESTIMATORS IN DOUBLE SAMPLING USING TWO AUXILIARY VARIABLES

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Abstract

In this paper, we have proposed a generalized class of double sampling estimators based on ratio type estimators for estimating the population mean of the study variable utilizing the available information in the form of known population parameter(s) of two auxiliary variables. The asymptotic expressions of bias and mean square error (MSE) of the proposed class of estimators have been obtained. A comparative study has been made with usual estimators available in the literature. The proposed class of estimators is found to be an improvement over chain ratio type estimator proposed by Chand (1975) and includes Sahai (1979) and Sen's (1978) estimators as special cases.

Keywords: Study Variable, Auxiliary Variable, Class of Double Estimators, Chain Ratio type Estimator, Bias, Mean Square Error.

1. Introduction

Let us consider a finite population $\underline{U} = (U_1, U_2, ..., U_N)$ of size N and let Y_i be the observation on the study variable y for the i^{th} unit of the population ($i = 1, 2, ..., N$). Also let X_i and Z_i be the observations on the auxiliary variable x and z respectively for the i th unit of the population U_i ($i = 1, 2, ..., N$). Further, let $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ and $\bar{Z} = \frac{1}{N}\sum_{i=1}^{N} Z_i$ be the population means of y, x and z respectively.

Let the two phase sampling scheme be such that

(i) The first phase sample s' of size n' ($n' < N$) is drawn from the population by SRSWOR to observe x and z only;

(ii) Given the first phase sample s', we select a second phase sample s ($s \subset s'$) of size n ($n < n'$) by SRSWOR.

Further let $\bar{x}$ and $\bar{y}$ be the sample means of the variables x and y respectively based on second phase sample s of size n; $\bar{x}'$ and $\bar{z}'$ the sample means of the variables x and z respectively based on first phase sample s' of size n'. In order to estimate the population mean $\bar{Y}$ of the study variable y and assuming that coefficient of variation C_z of variable z is known, we propose to use

$$\hat{\bar{Y}}_{SD} = \bar{y}\left\{\frac{\bar{x}'}{\bar{x}' + A(\bar{x} - \bar{x}')}\right\}\left\{\frac{(1-B)C_z + B\hat{C}'_z}{C_z}\right\}$$

$$\hat{\hat{Y}}_{SD} = \bar{y}\left\{\frac{\bar{x}'}{\bar{x}'+A(\bar{x}-\bar{x}')}\right\}\left\{1+B\,\frac{(\hat{C}'_z - C_z)}{C_z}\right\} \quad (1.1)$$

Where $C'_z = \frac{s'_z}{\bar{z}}$ is a consistent estimator of C_z and $s_z' = \sqrt{\frac{1}{n'-1}\sum_{i\in s'}^{n'}(Z_i-\bar{z}')^2}$ is the sample standard deviation of variable z. The use of coefficient of variation can be justified by the fact that the population coefficient of variation is a very stable quantity over time, see Murthy (1967), Sen (1978) etc. It may be noted that $\hat{\hat{Y}}_{SD}$ can be seen as the combination of Sahai's (1979) ratio type estimator and Sen (1978) estimator, which are also the special cases of the above mentioned estimator when $B=0$ and $A=0$ respectively. Also that $\hat{\hat{Y}}_{SD}$ Can be seen as a chain ratio type estimator on the lines of Chand (1975), Kiregyera (1980, 1984), Upadhyaya et al (1990), Singh and Singh (1991), Singh et al. (1994).

2. Bias and MSE Of $\hat{\hat{Y}}_{SD}$ under Double Sampling

In what follows the following notations are needed under the assumption that the population size N is so large that the sampling fractions $\frac{n}{N}$, $\frac{n'}{N}$ can be ignored and $N-1 \cong N$.

$\mu_{pqr} = \frac{1}{N}\sum_{i=1}^{N}(Y_i-\bar{Y})^p(X_i-\bar{X})^p(Z_i-\bar{Z})^r$, where p, q and r are non negative integers. $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$, $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$ and $S_z^2 = \frac{1}{N-1}\sum_{i=1}^{n}(Z_i-\bar{Z})^2$ are population variances of y, x and z respectively; $C_y^2 = \frac{S_y^2}{\bar{Y}^2} = \frac{\mu_{200}}{\bar{Y}^2}$, $C_x^2 = \frac{S_x^2}{\bar{X}^2} = \frac{\mu_{020}}{\bar{X}^2}$ and $C_z^2 = \frac{S_z^2}{\bar{Z}^2} = \frac{\mu_{002}}{\bar{Z}^2}$ are population coefficients of variation of y, x and z respectively.

Also $\rho_{xy} = \frac{\mu_{110}}{\sqrt{\mu_{020}\mu_{200}}}$ $\rho_{yz} = \frac{\mu_{101}}{\sqrt{\mu_{200}\mu_{002}}}$ $\rho_{xz} = \frac{\mu_{011}}{\sqrt{\mu_{020}\mu_{002}}}$

$\lambda_{yz} = \frac{\mu_{102}}{\bar{Y}.S_z^2}$, $\lambda_{xz} = \frac{\mu_{012}}{\bar{X}.S_z^2}$, $\beta_{1z} = \frac{\mu_{003}{}^2}{\mu_{002}{}^3}$ and $\beta_{2z} = \frac{\mu_{004}}{\mu_{002}{}^2}$

Suppose $\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $\bar{x}' = \bar{X}(1+e_1')$, $\bar{z}' = \bar{Z}(1+e_2')$ and $s'^2_z = S_z^2(1+e_3')$ such that

$$E(e_0) = E(e_1) = E(e_1') = E(e_2') = E(e_3') = 0 \quad (2.1)$$

Consider

$$\hat{\hat{Y}}_{SD} = \bar{y}\left\{\frac{\bar{x}'}{\bar{x}'+A(\bar{x}-\bar{x}')}\right\}\left\{\frac{(1-B)C_z + B\hat{C}'_z}{C_z}\right\}$$

$$= \bar{Y}(1+e_0)\left\{1+A\frac{(e_1-e_1')}{1+e_1'}\right\}^{-1}\left\{(1-B)+B(1+e_3')^{1/2}(1+e_2')^{-1}\right\}$$

$$= \bar{Y}(1+e_0)\left\{1-A(e_1-e_1')(1+e_1')^{-1}+A^2(e_1-e_1')^2(1+e_1')^{-2}+\ldots\right\}$$

$$\left\{(1-B)+B(1+\frac{1}{2}e_3'-\frac{1}{8}e_3'^2+\ldots)(1-e_2'+e_2'^2+\ldots)\right\}$$

$$= \bar{Y}(1+e_0)\{1-A(e_1-e_1^{'})(1-e_1^{'}+e_1^{'2}+...)+A^2(e_1^2+e_1^{'2}-2e_1e_1^{'})(1-2e_1^{'}+e_1^{'2}+...)+...\}$$
$$\left\{(1-B)+B(1-e_2^{'}+\frac{1}{2}e_3^{'}+e_2^{'2}-\frac{1}{8}e_3^{'2}-\frac{1}{2}e_2^{'}e_3^{'}+...)\right\}$$
$$= \bar{Y}(1+e_0)\{1-A(e_1-e_1^{'}-e_1e_1^{'}+e_1^{'2}+...)+A^2(e_1^2+e_1^{'2}-2e_1e_1^{'}+...)+...\}$$
$$\left\{(1-B)+B(1-e_2^{'}+\frac{1}{2}e_3^{'}+e_2^{'2}-\frac{1}{8}e_3^{'2}-\frac{1}{2}e_2^{'}e_3^{'}+...)\right\}$$
$$= \bar{Y}\{1+e_0-A(e_1-e_1^{'}-e_1e_1^{'}+e_1^{'2}+e_0e_1-e_0e_1^{'}+...)+A^2(e_1^2+e_1^{'2}-2e_1e_1^{'}+...)+...\}$$
$$\left\{(1-B)+B(1-e_2^{'}+\frac{1}{2}e_3^{'}+e_2^{'2}-\frac{1}{8}e_3^{'2}-\frac{1}{2}e_2^{'}e_3^{'}+...)\right\}$$
$$= \bar{Y}[1+e_0-A(e_1-e_1^{'}-e_1e_1^{'}+e'^2_1+e_0e_1-e_0e_1^{'})+A^2(e_1^2+e'^2_1-2e_1e_1^{'})$$
$$+AB(e_1e_1^{'}-e'_1\,e'_2-(1/2)e_1e'_3+(1/2)e'_1\,e'_3)$$
$$+B(-e'_2+(1/2)e'_3+e'^2_2-(1/8)e'^2_3-(1/2)e'_2\,e'_3+(1/2)e_0e'_3-e_0e'_2)+......]$$

(2.2)

As we know that

$$E(e_0^2)=\frac{C_y^2}{n},\ E(e_1^2)=\frac{C_x^2}{n},\ E(e_1^{'2})=E(e_1^{'}e_1^{'})=\frac{C_x^2}{n'},\ E(e_2^{'2})=\frac{C_z^2}{n'},$$
$$E(e_0e_1)=\frac{\rho_{xy}C_xC_y}{n},\ E(e_0e_1^{'})=\frac{\rho_{xy}C_xC_y}{n'},\ E(e_0e_1^{'})=\frac{\rho_{xy}C_xC_y}{n'},\ E(e_1^{'}e_2^{'})=E(e_1e_2^{'})=\frac{\rho_{xz}C_xC_z}{n'},$$
$$E(e_3^{'2})=\frac{(\beta_{2z}-1)}{n'},\ E(e_0e_3^{'})=\frac{\lambda_{yz}}{n'},\ E(e_1e_3^{'})=E(e_1^{'}e_3^{'})=\frac{\lambda_{xz}}{n'}\text{ and }E(e_2^{'}e_3^{'})=\frac{1}{n'}\sqrt{\beta_{1z}}C_z \quad (2.3)$$

Taking expectation of (2.2) and using (2.3), we have

$$E\left(\hat{\hat{Y}}_{SD}\right)=\bar{Y}\left[1-A\left\{-E(e_1e_1^{'})+E(e_1^{'2})+E(e_0e_1^{'})-E(e_0e_1^{'})\right\}+A^2\left\{E(e_1^{'2})+E(e_1^{'2})-2E(e_1e_1^{'})\right\}\right.$$
$$\left.+B\left\{E\left(e_2^{'2}\right)-\frac{1}{8}E(e_3^{'2})-\frac{1}{2}E(e_2^{'}e_3^{'})+\frac{1}{2}E(e_0e_3^{'})-E(e_0e_2^{'})\right\}+AB\left\{E(e_1e_2^{'})-E(e_1^{'}e_2^{'})-\frac{1}{2}E(e_1^{'}e_3^{'})+\frac{1}{2}E(e_1^{'}e_3^{'})\right\}\right]$$
$$=\bar{Y}\left[1+\left(\frac{1}{n}-\frac{1}{n'}\right)(A^2C_x^2-A\rho_{xy}C_xC_y)+\frac{B}{n'}\left\{C_z^2-\frac{1}{8}(\beta_{2z}-1)-\frac{1}{2}\sqrt{\beta_{1z}}C_z+\frac{1}{2}\lambda_{yz}-\rho_{yz}C_yC_z\right\}\right]$$

(2.4)

$$\neq \bar{Y}$$

showing that the proposed estimator is a biased estimator of population mean $\bar{Y}$ and its bias is given by

$$B(\hat{\hat{Y}}_{SD})=E(\hat{\hat{Y}}_{SD})-\bar{Y}$$
$$=\bar{Y}\left[\left(\frac{1}{n}-\frac{1}{n'}\right)(A^2C_x^2-A\rho_{xy}C_xC_y)+\frac{B}{n'}\left\{C_z^2-\frac{1}{8}(\beta_{2z}-1)-\frac{1}{2}\sqrt{\beta_{1z}}C_z+\frac{1}{2}\lambda_{yz}-\rho_{yz}C_yC_z\right\}\right] \quad (2.5)$$

Considering the mean square error of the proposed estimator $\overline{Y_{SD}}$ by using (2.2) we have

$$MSE(\hat{\hat{Y}}_{SD})=E(\hat{\hat{Y}}_{SD}-\bar{Y})^2$$
$$=\bar{Y}^2E\left\{e_0-A(e_1-e_1^{'})+B(-e_2^{'}+\frac{1}{2}e_3^{'})\right\}^2 \text{ (to the first order of approximation)}$$

$$= \bar{Y}^2[E(e_0^2) + A^2\{E(e_1^2) + E(e'^2_1) - 2E(e_1 e_1')\} + B^2\{E(e'^2_2) + (1/4)E(e'^2_3) - E(e'_2\, e'_3)\}$$
$$- 2A\{E(e_0 e_1) - E(e_0 e'_1)\} - 2B(E(e_0 e_2') - (1/2)E(e_0 e_3')\}$$
$$+ 2AB\{E(e_1 e_2') - E(e'_1\, e'_2) - (1/2)E(e_1 e'_3) + (1/2)E(e'_1\, e'_3)\}]$$
$$= \bar{Y}^2\left[\frac{C_y^2}{n} + \left(\frac{1}{n} - \frac{1}{n'}\right)(A^2C_x^2 - 2A\rho_{xy}C_xC_y) + \frac{B^2}{n'}C_z^2 + \frac{B^2}{4n'}(\beta_{2z} - 1) - \frac{B^2}{n'}\sqrt{\beta_{1z}}C_z - 2\frac{B}{n'}\rho_{yz}C_yC_z + \frac{B}{n'}\lambda_{yz}\right]$$
$$= \bar{Y}^2\left[\frac{C_y^2}{n} + \left(\frac{1}{n} - \frac{1}{n'}\right)(A^2C_x^2 - 2A\rho_{xy}C_xC_y) + \frac{B^2}{4n'}\left\{(\beta_{2z} - \beta_{1z} - 1) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\} + \frac{B}{n'}\left(\lambda_{yz} - 2\rho_{yz}C_yC_z\right)\right] \tag{2.6}$$

The optimizing values of the characterizing scalars are given by

$$A_{opt} = \rho_{xy}\frac{C_y}{C_x} = A'(say) \text{ and } B_{opt} = \frac{2\left(2\rho_{yz}C_yC_z - \lambda_{yz}\right)}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}} = B'(say) \tag{2.7}$$

The minimum mean square error, under the optimizing values of the characterizing scalars, is given by

$$MSE(\hat{\hat{Y}}_{SD})_{\min} = \bar{Y}^2\left[\frac{1}{n}(1-\rho_{xy}^2)C_y^2 + \frac{\rho_{xy}^2C_y^2}{n'} - \frac{1}{n'}\frac{(\lambda_{yz} - 2\rho_{yz}C_yC_z)^2}{\{(\beta_{2z} - \beta_{1z} - 1) + (2C_z - \sqrt{\beta_{1z}})^2\}}\right] \tag{2.8}$$

3. Class of Estimators based on the Estimated Values of the Characterizing Scalars

The optimum values of the characterizing scalars are rarely known in practice, hence they may be estimated by estimators based on the sample data. The optimizing values of the characterizing scalars can be written as

$$A_{opt} = \rho_{xy}\frac{C_y}{C_x} = \frac{S_{xy}}{S_x^2}\frac{\bar{X}}{\bar{Y}} = A'$$

$$B_{opt} = \frac{2\left(2\rho_{yz}C_yC_z - \lambda_{yz}\right)}{\left\{\left(\beta_{2z} - 1\right) + 4C_z^2 - 4\sqrt{\beta_{1z}}C_z\right\}} = \frac{2\left(2\frac{S_{yz}}{\bar{Y}.\bar{Z}} - \frac{\mu_{102}}{\bar{Y}.S_z^2}\right)}{\left\{4\left(\frac{S_z^2}{\bar{Z}^2} - \frac{\mu_{003}}{\mu_{002}}\right) + \left(\frac{\mu_{004}}{\mu_{002}^2} - 1\right)\right\}} = B'$$

We may take $\hat{A}'$ and $\hat{B}'$ as estimators of A' and B' such that

$$\hat{A}' = \frac{\hat{\mu}_{110}}{\hat{\mu}_{020}}\frac{\bar{x}}{\bar{y}}$$

$$\hat{B}' = \frac{2\left(2\frac{\hat{\mu}_{101}}{\bar{y}.\bar{z}} - \frac{\hat{\mu}_{102}}{\bar{y}.\hat{\mu}_{002}}\right)}{\left\{4\left(\frac{\hat{\mu}_{002}}{\bar{z}^2} - \frac{\hat{\mu}_{003}}{\hat{\mu}_{002}}\right) + \left(\frac{\hat{\mu}_{004}}{\hat{\mu}_{002}^2} - 1\right)\right\}} \tag{3.1}$$

where $\hat{\mu}_{101} = m_{101}$, $\hat{\mu}_{110} = m_{110}$, $\hat{\mu}_{020} = m_{020}$, $\hat{\mu}_{002} = m_{002}$, $\hat{\mu}_{102} = \frac{n.m_{102}}{(n-2)}$, $\hat{\mu}_{003} = \frac{n.m_{003}}{(n-2)}$ and $\hat{\mu}_{004} = \frac{n^2.m_{004} - 3(2n-1)S_x^4}{(n^2 - 3n + 3)}$ are the estimators with their expected values μ_{101}, μ_{110}, μ_{020}, μ_{002}, μ_{102}, μ_{003} and μ_{004} respectively (for large value of population size N in case of

SRSWOR) such that $m_{pqr} = \frac{1}{(n-1)}\sum_{i\in s}^{n}(y_i-\bar{y})^p(x_i-\bar{x})^p(z_i-\bar{z})^r$. And let us denote $\hat{\mu}_{110} = \mu_{110}(1+e_4), \hat{\mu}_{020} = \mu_{020}(1+e_5), \hat{\mu}_{101} = \mu_{101}(1+e_6), \hat{\mu}_{102} = \mu_{102}(1+e_7)$
$\hat{\mu}_{002} = \mu_{002}(1+e_8)$, $\hat{\mu}_{003} = \mu_{003}(1+e_9)$, $\hat{\mu}_{004} = \mu_{004}(1+e_{10})$ such that $E(e_i)=0, \forall i = 4,5,...,10$ (3.2)

The proposed estimator under the estimated optimum values $\hat{A}'$ *and* $\hat{B}'$ takes form

$$\hat{\hat{Y}}^*_{SD} = \bar{y}\left\{\frac{\bar{x}'}{\bar{x}'+\hat{A}'(\bar{x}-\bar{x}')}\right\}\left\{\frac{(1-\hat{B}')C_z+\hat{B}'\hat{C}'_z}{C_z}\right\} \tag{3.3}$$

Putting the values from (3.2) in (3.3) and simplifying we have

$$\hat{\hat{Y}}^*_{SD} = \bar{Y}\left\{1+e_0-A'(e_1-e_1')+B'\left(-e_2'+\frac{1}{2}e_3'\right)\right\}+O(e^2) \tag{3.4}$$

Now the mean square error of the proposed estimator $\hat{\hat{Y}}^*_{SD}$ under the estimated optimum values of the characterizing scalar is given by

$$MSE(\hat{\hat{Y}}^*_{SD}) = E(\hat{\hat{Y}}^*_{SD}-\bar{Y})^2$$

$$= \bar{Y}^2E\left\{e_0-A'(e_1-e_1')+B'(-e_2'+\frac{1}{2}e_3')\right\}^2 \text{ (to the first order of approximation)}$$

$$=\bar{Y}^2\Big[E(e_0^2)+A'^2\left\{E(e_1^2)+E(e_1'^2)-2E(e_1e_1')\right\}+B'^2\left\{E(e_2'^2)+\frac{1}{4}E(e_3'^2)-E(e_2'e_3')\right\}-2A'\left\{E(e_0e_1)-E(e_0e_1')\right\}-2B'\left\{E(e_0e_2')-\frac{1}{2}E(e_0e_3')\right\}$$
$$+2A'B'\left\{E(e_1e_2')-E(e_1'e_2')-\frac{1}{2}E(e_1e_3')+\frac{1}{2}E(e_1'e_3')\right\}\Big]$$

$$=\bar{Y}^2\left[\frac{C_y^2}{n}+\left(\frac{1}{n}-\frac{1}{n'}\right)(A'^2C_x^2-2A'\rho_{xy}C_xC_y)+\frac{B'^2}{4n'}\{(\beta_{2z}-\beta_{1z}-1)+(2C_z-\sqrt{\beta_{1z}})^2\}+\frac{B'}{n'}(\lambda_{yz}-2\rho_{yz}C_yC_z)\right]$$

$$=\bar{Y}^2\left[\frac{1}{n}(1-\rho_{xy}^2)C_y^2+\frac{\rho_{xy}^2C_y^2}{n'}-\frac{1}{n'}\frac{(\lambda_{yz}-2\rho_{yz}C_yC_z)^2}{\left\{(\beta_{2z}-\beta_{1z}-1)+\left(2C_z-\sqrt{\beta_{1z}}\right)^2\right\}}\right]$$

$$= MSE(\hat{\hat{Y}}_{SD})_{\min} \tag{3.5}$$

which is same as minimum mean square error of $\hat{\hat{Y}}_{SD}$ obtained in (2.8).

4. Comparison with various commonly used Estimators

The sample mean $\bar{y}$ has the variance

$$V(\bar{y}) = \frac{\bar{Y}^2C_y^2}{n} \tag{4.1}$$

The double sampling ratio estimator $\bar{y}_{Rd} = \frac{\bar{y}}{\bar{x}}\bar{x}'$ has the mean square error given by

$$MSE(\bar{y}_{Rd}) = \bar{Y}^2\left\{\frac{C_y^2}{n}+\left(\frac{1}{n}-\frac{1}{n'}\right)(C_x^2-2\rho_{xy}C_xC_y)\right\} \tag{4.2}$$

The chain ratio type estimator of Chand (1975) utilizing auxiliary information about two auxiliary variable is $\bar{y}_{RC} = \bar{y}\frac{\bar{x}'}{\bar{x}}\frac{\bar{Z}}{\bar{z}'}$ having mean square error given by

$$MSE(\bar{y}_{RC}) = \bar{Y}^2 \left\{ \left(\frac{1}{n} - \frac{1}{n'} \right)(C_x^2 - 2\rho_{xy} C_x C_y) + \frac{1}{n}(C_y^2 + C_z^2 - 2\rho_{yz} C_y C_z) \right\} \tag{4.3}$$

The double sampling regression estimator is $\bar{y}_{LRd} = \bar{y} + \frac{s_{xy}}{s_x^2}(\bar{x}' - \bar{x})$ having mean square error given by

$$MSE(\bar{y}_{LRd}) = \frac{\bar{Y}^2}{n}(1-\rho_{xy}^2)C_y^2 + \frac{\bar{Y}^2 \rho_{xy}^2 C_y^2}{n'} \tag{4.4}$$

Further, the minimum mean square error of the proposed class of estimators (or mean square error of the estimator based on the estimated optimum values) is given by

$$MSE(\bar{Y}_{SD})_{\min} = MSE(\bar{Y}_{SD}^*) = \bar{Y}^2 \left[\frac{1}{n}(1-\rho_{xy}^2)C_y^2 + \frac{\rho_{xy}^2 C_y^2}{n'} - \frac{1}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}} \right]$$

$$= M\,(say) \tag{4.5}$$

Comparing (4.1) and (4.5) we have

$$V(\bar{y}) - M = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right) \rho_{xy}^2 C_y^2 + \frac{1}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}} \ge 0 \tag{4.6}$$

Comparing (4.2) and (4.5) we have

$$MSE(\bar{y}_{Rd}) - M = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right)(\rho_{xy}^2 C_y^2 + C_x^2 - 2\rho_{xy} C_x C_y) + \frac{1}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}}$$

$$= \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right)(C_x - \rho_{xy} C_y)^2 + \frac{1}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}} \ge 0 \tag{4.7}$$

Comparing (4.3) and (4.5) we have

$$MSE(\bar{y}_{RC}) - M = \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right)(\rho_{xy}^2 C_y^2 + C_x^2 - 2\rho_{xy} C_x C_y) + \frac{1}{n}(C_z^2 - 2\rho_{yz} C_y C_z)$$

$$+ \frac{1}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}}$$

$$= \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right)(C_x - \rho_{xy} C_y)^2 + \frac{1}{n}(C_z^2 - 2\rho_{yz} C_y C_z) + \frac{1}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}} \ge 0$$

provided $\rho_{yz} \le \frac{1}{2}\frac{C_z}{C_y}$ (4.8)

Comparing (4.4) and (4.5) we have

$$MSE(\bar{y}_{LRd}) - M = \frac{\bar{Y}^2}{n'} \frac{\left(\lambda_{yz} - 2\rho_{yz} C_y C_z\right)^2}{\left\{\left(\beta_{2z} - \beta_{1z} - 1\right) + \left(2C_z - \sqrt{\beta_{1z}}\right)^2\right\}} \ge 0 \tag{4.9}$$

5. Conclusion

The comparative study shows that the proposed estimator $\hat{\bar{Y}}_{SD}$ as well as estimator $\hat{\bar{Y}}_{SD}^*$ based on the estimated optimum values of the characterizing scalars establish their superiority over sample mean $\bar{y}$, double sampling ratio estimator $\bar{y}_{Rd}$ and double sampling linear regression estimator $\bar{y}_{LRd}$ (c.f. (4.6), (4.7) and (4.9) respectively). Also the comparison with Chand's chain ratio type estimator (c.f. (4.8)) shows that the proposed estimators are better provided the condition $\rho_{yz} \leq \frac{1}{2}\frac{C_z}{C_y}$ is satisfied.

6. References

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