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ON THE COMBINATION OF ESTIMATORS OF FINITE POPULATION MEAN USING INCOMPLETE MULTI-AUXILIARY INFORMATION

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ABSTRACT

The present study is concerned with the estimation of a finite population mean when we have information on several auxiliary variables only for some part of the population. The maximum utilization of incomplete multi-auxiliary information is carried out in such cases by stratifying the population on the basis of available multi-auxiliary information at hand. This paper deals with the situation where some of the auxiliary variables are positively and some of them are negatively correlated with the main variable. For this purpose, in this paper ratio-cum-product and regression-cum-product type estimators are considered for estimating the mean of the finite population utilizing available incomplete multi-auxiliary information. The approximate expressions for bias and mean square error of the suggested estimators have also been derived and theoretical results are numerically supported.

Key Words: Bias; Mean Square Error; Incomplete Multi-Auxiliary Information.

1. Introduction

The use of supplementary information is widely discussed in sampling theory. Auxiliary variables are commonly used in sample survey practices to obtain improved designs and to achieve higher precision in the estimates of some population parameters such as the mean or the total of a study variable. In this context it is well known that when the auxiliary information is to be used at the estimation stage, the ratio, product and regression methods are widely employed in many situations. It should be recalled that ratio and regression estimators are used when study variable is highly positively correlated with the auxiliary variable and product estimator is deployed when the study variable is negatively correlated with the auxiliary variable.

There may arise situations where we possess information about several auxiliary variables but each variable may be known for some part of population only. To handle such types of situations, Singh (1977), Srivastava and Garg (2009; 2013) have considered the concept of stratification for weighting the given incomplete auxiliary information, which enables the investigator to make maximum use of such information. In this technique the population may be considered as consisting of different strata according to the number of known auxiliary variables. Then a stratified sample of size 'n' can be selected from each stratum of the population.

It may happen that some of the auxiliary variables are positively and some are negatively correlated with the study variable. Thus, in order to obtain a more precise estimate of the population mean, the aim of this paper is to develop ratio-cum-product and regression-cum-product type estimators by making maximum use of available incomplete multi-auxiliary information. It is seen that it is always better to use additional auxiliary variables, which are correlated with Y. These estimators are constructed when frame is known for each stratum i.e. information on number of auxiliary variables that can be collected for each and every population unit should be known in advance.

Generally the stratification is done on the basis of heterogeneity in the population with respect to the study variable Y. But we view stratification in the other way. In our case, the heterogeneity of the population is considered with respect to the unequal number of auxiliary variables. We stratify the population in terms of the information provided by the p auxiliary variables. Thus, there will be a stratum for which no auxiliary information is available, p strata for which only one auxiliary variable out of p auxiliary variables is known. Similarly there will be ${}^{p}C_{2}$ strata for which the two auxiliary variables are known, ${}^{p}C_{3}$ strata for which three auxiliary variables are known, and so on. Ultimately, we will have a stratum for which all the p auxiliary variables are known. It is seen that the given method, considering stratification of population on the basis of unequal number of auxiliary variable is capable of giving more precise results than simple mean per unit.

In Section 3 of the paper the construction of strata is explained while the suggested ratio-cum-product type estimator and regression-cum-product type estimator are discussed in section 4 and 5 respectively. In section 6, an empirical study is carried out.

2. Notations

Let us consider a finite population U = ($U_1, U_2, \ldots, U_N$) of N identifiable units taking values on a study variable Y and p auxiliary variables $X_1, X_2, \ldots, Xp$, which are correlated with Y. Auxiliary variables $X_1, X_2, \ldots, Xp$ are known for total $M_1, M_2, \ldots, Mp$ units of the population respectively. For the maximum utilization of available incomplete auxiliary information, the population is divided into different strata according to the known number of auxiliary variables and a random sample of n units is drawn from these groups with simple random sampling without replacement.

${}^{p}C_{j}$: Number of strata for which j auxiliary variables are known; $j = 0, 1, 2, \ldots, p$.

N : Population size

n : Sample size

N_0 : Size of the stratum for which no auxiliary variable is known

N_{ii} : Size of the stratum for which 1 auxiliary variable X_i is known; $i = 1,2, \ldots, p$

N_{ij} : size of the stratum for which 2 auxiliary variable X_i and X_j are known; $i < j;\ i, j = 1, 2, \ldots, p$

N_{ijk} : size of the stratum for which 3 auxiliary variable X_i, X_j and X_k are known; $i < j < k; i, j, k = 1,2, \ldots, p$

$N_{1,2,\ldots\ldots,p}$: size of the strata for which all p auxiliary variable $X_1, X_2, \ldots, X_p$ is known; $i < j < k; i, j, k = 1,2,\ldots, p$
Where $N_{ii} + N_{ij} + N_{ijk} + \ldots + N_{1,2,\ldots i \ldots,p} = M_i$

2^p : Total number of strata, i.e. $\sum_{i=0}^{p} {}^{p}C_i = 2^p$

N_i : population size of the i^th stratum; $i = 1, 2, \ldots, 2^p$ such that $\sum_{i=1}^{2^p} N_i = N$

n_i : sample size of the i^th stratum; $i = 1, 2, \ldots, 2^p$ such that $\sum_{i=1}^{2^p} n_i = n$

Y_{ik} : Value of the k^th observation on variable under study in i^th stratum; $i = 1, 2, \ldots, {}^{p}C_j$; $j = 0, 1, 2, \ldots, p$; $k = 1, 2, \ldots, N_i$

X_{ijk} : Value of the k^th observation on j^th auxiliary variable in i^th stratum $i = 1, 2, \ldots, {}^{p}C_j$; $j = 0, 1, 2, \ldots, p$; $k = 1, 2, \ldots, N_i$

$\overline{Y}_i$: Population mean of the Y variable in i^th stratum

$\overline{y}_i$: Sample mean of the Y variable in i^th stratum

$\overline{X}_{ij}$: Population mean of the j^th auxiliary variable in i^th stratum

$\overline{x}_{ij}$: Sample mean of the j^th auxiliary variable in i^th stratum

S_i^2 : Population mean square error of Y variable in i^th stratum

S_{ij}^2 : Population mean square error of j^th auxiliary variable in i^th stratum

C_i^2 : Coefficient of variation of the variable under study Y in i^th stratum, i.e. $C_i^2 = \frac{S_i^2}{\overline{Y}_i^2}$

C_{ij}^2 : Coefficient of variation of the j^th auxiliary variable in i^th stratum, i.e. $C_{ij}^2 = \frac{S_{ij}^2}{\overline{X}_{ij}^2}$

ρ_{ij} : Correlation coefficient between the variables Y, variable under study and X_{ij}, j^th auxiliary variable in i^th stratum

ρ_{ijh} : Correlation coefficient between the variables X_j and X_h ($j \neq h$) in i^th stratum.

b_{ij} : Regression coefficient of the variables Y, variable under study and X_{ij}, j^{th} auxiliary variable in i^{th} stratum

W_i : Proportion of units in the i^{th} stratum, i.e. $W_i = \frac{N_i}{N}$

f : Sampling fraction, i.e. $f = \frac{n}{N}$

f_i: Sampling fraction in the i^{th} stratum, i.e. $f_i = \frac{n_i}{N_i}$

α_{ij} : Weights, attached to the j^{th} auxiliary variable in i^{th} stratum adding up to unity i.e. $\sum_{j=1}^{p} \alpha_{ij} = 1$

Define

$$e_{io} = \left(\frac{\bar{y} - \bar{Y}}{\bar{Y}} \right) \text{ and } \quad e_{ij} = \left(\frac{\bar{x}_{ij} - \bar{X}_{ij}}{\bar{X}_{ij}} \right); i < j;\ i, j = 1, 2, \ldots, p$$

Such that

$$E(e_{i0}) = E(e_{ij}) = 0, \qquad j = 1, 2, \ldots, p$$

$$E(e_{io}^2) = \left(\frac{1 - f_i}{n_i} \right) C_{yi}^2, \quad E(e_{ij}^2) = \left(\frac{1 - f_i}{n_i} \right) C_{xij}^2, j = 1, 2, \ldots, p$$

$$E(e_{io} e_{ij}) = \left(\frac{1 - f_i}{n_i} \right) \rho_{ij} C_{yi} C_{xij}, \qquad j = 1, 2, \ldots, p$$

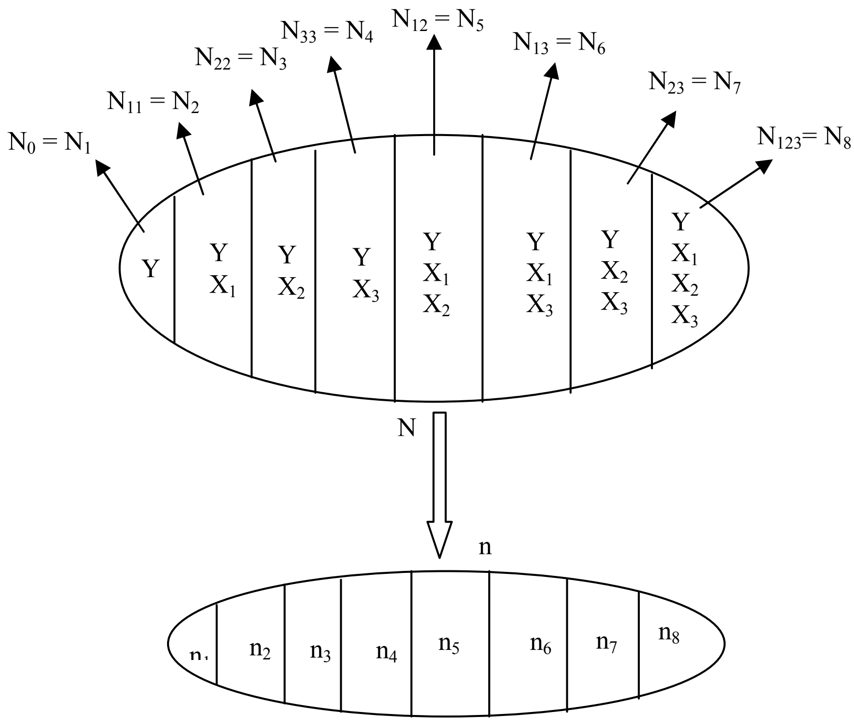
$$E(e_{ij} e_{ik}) = \left(\frac{1 - f_i}{n_i} \right) \rho_{ijk} C_{xij} C_{xik}, \qquad j = 1, 2, \ldots, p$$

$$K_{ij} = \rho_{ij} \frac{C_i}{C_{ij}}; \quad j=1,2, \ldots, p$$

3. The construction of strata

The following figure shows the construction of strata ($2^3 = 8$) on the basis of available incomplete three-auxiliary information (X_1, X_2 and X_3) and then selection of sample:

If some of the ρ_{ij} 's are positive; say, for $j = 1,2, ..., p'$ and some ρ_{ij} 's are negative; say, for $j = p'+1, ..., p \ \forall i$ then our estimator will be weighted ratio-cum-product type or regression-cum-product type. Now, we have obtained the following weighted ratio-cum-product type or regression-cum-product type estimators using incomplete multi-auxiliary information:



4. Ratio-Cum-Product Type Estimator

$$\bar{y}_{pr.ratcp} = \sum_{i=1}^{2^p} W_i \left[\sum_{j=1}^{p'} \alpha_{ij} g_{ij.rat}\left(y_i, x_{ij}\right) + \sum_{j=p'+1}^{p} \alpha_{ij} g_{ij.prod}\left(y_i, x_{ij}\right) \right]$$

Where, $g_{ij.rat}(y_i, x_{ij}) = \dfrac{\bar{y}_i}{\bar{x}_{ij}} \bar{X}_{ij}$ *and* $g_{ij.prod}(y_i, x_{ij}) = \dfrac{\bar{y}_i}{\bar{X}_{ij}} \bar{x}_{ij}$

4.1 Its Bias and MSE

To the first-degree approximation, the bias and mean square error are respectively given by

$$E(\bar{y}_{pr.ratcp}) = E\left[\sum_{i=1}^{2^p} W_i \left\{ \sum_{j=1}^{p'} \alpha_{ij} g_{ij.rat}(y_i, x_{ij}) + \sum_{j=p'+1}^{p} \alpha_{ij} g_{ij.prod}(y_i, x_{ij}) \right\}\right]$$

$$= \bar{Y} + \sum_{i=1}^{2^p} W_i \bar{Y}_i \frac{1-f_i}{n_i}\left[\sum_{j=1}^{p'} \alpha_{ij} C_{ij}^2 - \left\{\sum_{j=1}^{p} \alpha_{ij} C_{ij}^2 K'_{ij}\right\}\right]$$

Where $K'_{ij} = K_{ij}$, for $j = 1, 2, \ldots, p'$

$K'_{ij} = -K_{ij}$, for $j = p' + 1, p' + 2, \ldots, p$

$$Bias(\bar{y}_{pr.ratcp}) = E(\bar{y}_{pr.ratcp}) - \bar{Y}$$

$$= \sum_{i=1}^{2^p} W_i \bar{Y}_i \frac{1-f_i}{n_i}\left[\sum_{j=1}^{p'} \alpha_{ij} C_{ij}^2 - \sum_{j=1}^{p} \alpha_{ij} C_{ij}^2 K'_{ij}\right]$$

$$MSE(\bar{y}_{pr.ratcp}) = MSE\left[\sum_{i=1}^{2^p} W_i \left\{ \sum_{j=1}^{p'} \alpha_{ij} g_{ij.rat}(y_i, x_{ij}) + \sum_{j=p'+1}^{p} \alpha_{ij} g_{ij.prod}(y_i, x_{ij}) \right\}\right] + 0$$

$$= \sum_{i=1}^{2^p} W_i^2 MSE\left[\sum_{j=1}^{p'} \alpha_{ij} g_{ij.rat}(y_i, x_{ij}) + \sum_{j=p'+1}^{p} \alpha_{ij} g_{ij.prod}(y_i, x_{ij})\right] + 0$$

The covariance terms will vanish because all strata are independent of each other.

$$= \sum_{i=1}^{2^p} W_i^2 \left[\sum_{j=1}^{p'} \sum_{h=1}^{p'} \alpha_{ij} \alpha_{ih} \operatorname{cov}(g_{ij.rat}, g_{ij.rat}) + \sum_{j=p'+1}^{p} \sum_{h=p'+1}^{p} \alpha_{ij} \alpha_{ih} \operatorname{cov}(g_{ij.prod}, g_{ij.prod}) + \sum_{j=1}^{p'} \sum_{h=p'+1}^{p} \alpha_{ij} \alpha_{ih} \operatorname{cov}(g_{ij.rat}, g_{ij.prod}) \right]$$

Where, $\left(\frac{1}{n_i} - \frac{1}{N_i}\right) v_{ijh} = Cov(g_{ij}, g_{ih})$

$$= \sum_{i=1}^{2^p} W_i^2 \frac{1-f_i}{n_i}\left[\sum_{j=1}^{p} \sum_{h=1}^{p} \alpha_{ij} \alpha_{ih} v_{ijh}\right]$$

where,

$$v_{ijh} = v_{ijh.rat} = \overline{Y}_i^{\,2}\left[C_i^2 + \rho_{ijh}C_{ij}C_{ih} - \rho_{ij}C_iC_{ij} - \rho_{ih}C_iC_{ih}\right]$$

$$For \quad j = 1,2,..........,p' \quad and \quad h = 1,2,..........,p'$$

$$v_{ijh} = v_{ijh.prod} = \overline{Y}_i^{\,2}\left[C_i^2 + \rho_{ijh}C_{ij}C_{ih} + \rho_{ij}C_iC_{ij} + \rho_{ih}C_iC_{ih}\right]$$

$$For \quad j = p'+1, p'+2,..........,p \quad and \quad h = p'+1, p'+2,..........,p$$

$$v_{ijh} = v_{ijh.ratcp} = \overline{Y}_i^{\,2}\left[C_i^2 - \rho_{ijh}C_{ij}C_{ih} - \rho_{ij}C_iC_{ij} + \rho_{ih}C_iC_{ih}\right]$$

$$For \quad j = 1,2,..........,p' \quad and \quad h = p'+1, p'+2,..........,p$$

Remark 1 : The optimum values of α_{ij} for $j = 1, 2, ..., p$ are obtained by adopting the procedure given by Olkin(1958).

Optimum Values of α_{ij} for j = 1, 2,………, p
It is fairly simple to establish that the optimum α_{ij} is given by

$$\alpha_{ij} = \frac{Sum\ of\ the\ elements\ of\ the\ j^{th}\ column\ of\ V_i^{-1}}{Sum\ of\ all\ the\ p^2\ elements\ in\ V_i^{-1}}$$

Where the matrix $V_i=(v_{ijh})$ and $\underset{\sim}{\alpha_i} = \left(\alpha_{i1}, \alpha_{i2}, ..., \alpha_{ip}\right)$, $\underset{\sim}{\alpha_i}'$ being the transpose of $\underset{\sim}{\alpha_i}$. V_i^{-1} is the matrix inverse to V_i. Using the optimum weights, the mean square error is found to be

$$\mathrm{MSE}\left(\overline{y}_{pr.ratcp}\right) = \sum_{i=1}^{2^p} W_i \left[\left(\frac{1}{n_i} - \frac{1}{N_i}\right) \Big/ \text{Sum of all the } p^2 \text{ elements in } V_i^{-1}\right]$$

5. Regression-Cum-Product Type Estimator

$$\overline{y}_{pr.regcp} = \sum_{i=1}^{2^p} W_i \left[\sum_{j=1}^{p'} \alpha_{ij}\, g_{ij.reg}\left(y_i, x_{ij}\right) + \sum_{j=p'+1}^{p} \alpha_{ij}\, g_{ij.prod}\left(y_i, x_{ij}\right)\right]$$

Where, $\quad g_{ij.reg}(y_i, x_{ij}) = \overline{y}_i + b_{ij}\left(\overline{X}_{ij} - \overline{x}_{ij}\right)$

$$and \quad g_{ij.prod}(y_i, x_{ij}) = \frac{\overline{y}_i}{\overline{X}_{ij}}\overline{x}_{ij}$$

5.1 Its Bias and MSE

To the first-degree approximation, the bias and mean square error are respectively given by

$$E\left(\overline{y}_{pr.regcp}\right) = E\left[\sum_{i=1}^{2^p} W_i \left\{\sum_{j=1}^{p'} \alpha_{ij}\, g_{ij.reg}\left(y_i, x_{ij}\right) + \sum_{j=p'+1}^{p} \alpha_{ij}\, g_{ij.prod}\left(y_i, x_{ij}\right)\right\}\right]$$

$$= \bar{Y} + \sum_{i=1}^{2^p} W_i \bar{Y}_i \frac{1-f_i}{n_i}\left[\sum_{j=p'+1}^{p} \alpha_{ij} C_{ij}^2 K_{ij}\right]$$

$$Bias\left(\bar{y}_{pr.regcp}\right) = E\left(\bar{y}_{pr.regcp}\right) - \bar{Y}$$

$$= \sum_{i=1}^{2^p} W_i \bar{Y}_i \frac{1-f_i}{n_i}\left[\sum_{j=p'+1}^{p} \alpha_{ij} C_{ij}^2 K_{ij}\right]$$

$$MSE\left(\bar{y}_{pr.regcp}\right) = MSE\left[\sum_{i=1}^{2^p} W_i \left\{\sum_{j=1}^{p'} \alpha_{ij} g_{ij.reg}\left(y_i, x_{ij}\right) + \sum_{j=p'+1}^{p} \alpha_{ij} g_{ij.prod}\left(y_i, x_{ij}\right)\right\}\right] + 0$$

$$= \sum_{i=1}^{2^p} W_i^2 MSE\left[\sum_{j=1}^{p'} \alpha_{ij} g_{ij.reg}\left(y_i, x_{ij}\right) + \sum_{j=p'+1}^{p} \alpha_{ij} g_{ij.prod}\left(y_i, x_{ij}\right)\right] + 0$$

The covariance terms will vanish because all strata are independent of each other.

$$= \sum_{i=1}^{2^p} W_i^2 \frac{1-f_i}{n_i}\left[\sum_{j=1}^{p}\sum_{h=1}^{p} \alpha_{ij}\alpha_{ih} v_{ijh}\right]$$

Where,

$$v_{ijh} = v_{ijh.reg} = \left[S_i^2 + b_{ij} b_{ih} \rho_{ijh} S_{ij} S_{ih} - b_{ij} \rho_{ij} S_i S_{ij} - b_{ih} \rho_{ih} S_i S_{ih}\right]$$

$$For \quad j = 1, 2, \ldots\ldots\ldots, p' \quad and \quad h = 1, 2, \ldots\ldots\ldots, p'$$

$$v_{ijh} = v_{ijh.prod} = \bar{Y}_i^2\left[C_i^2 + \rho_{ijh} C_{ij} C_{ih} + \rho_{ij} C_i C_{ij} + \rho_{ih} C_i C_{ih}\right]$$

$$For \quad j = p'+1, p'+2, \ldots\ldots\ldots, p \quad and \quad h = p'+1, p'+2, \ldots\ldots\ldots, p$$

$$v_{ijh} = v_{ijh.regcp} = \bar{Y}_i\left[\bar{Y}_i C_i^2 - \bar{X}_{ij} b_{ij} \rho_{ijh} C_{ij} C_{ih} - \bar{X}_{ij} b_{ij} \rho_{ij} C_i C_{ij} + \bar{Y}_i \rho_{ih} C_i C_{ih}\right]$$

$$For \quad j = 1, 2, \ldots\ldots\ldots, p' \quad and \quad h = p'+1, p'+2, \ldots\ldots\ldots, p$$

Remark 2: It is to be noted that $\bar{y}'_{pr}$ could be used if the parameters b, C_y, C_x and ρ_{yx} are known. As remarked by Murthy (1967, p96), Sahai and Sahai (1985) and Tracy and Singh (1999), these parameters are stable quantities, therefore, these can be known either from the past studies or from the experience gathered in due course of time.

6. An Empirical Study

We consider the population of size N=90 for comparing the proposed ratio-cum-product and regression-cum-product estimators with mean per unit estimator. The population set consists of the data where some of the auxiliary variables are positively correlated and others are negatively related with the study variable. Suppose a sample of size n=43 is drawn by SRSWOR from this population.

The computed sample size for each stratum by proportional allocation is given below:

Strata	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**Total**
N_i	16	13	7	10	12	8	9	15	90
n_i	8	6	3	5	6	4	4	7	43

Table 1: The computed strata sample sizes using proportional allocation

Without Stratification	$\bar{Y}$ = 48.14444, S_y^2 = 228.01261,
Stratum	
I	$\bar{Y}_1$=45.81250 S_1^2=207.22917
II	$\bar{Y}_2$=48.53846 $\bar{X}_{21}$=51.69231 K_{21}=0.57466 S_2^2=284.26923 ρ_{21}=0.85959 b_{21}=0.53960 S_{21}^2=721.39744
III	$\bar{Y}_3$=47.85714 $\bar{X}_{32}$=54.57143 K_{32}= - 0.70742 S_3^2=285.14286 ρ_{32} = - 0.87272 S_{32}^2=564.28571
IV	$\bar{Y}_4$=50.30000 $\bar{X}_{43}$=46.60000 K_{43}=0.64864 S_4^2=312.23333 ρ_{43} =0.93639 b_{43}=0.70014 S_{43}^2=558.48889
V	$\bar{Y}_5$=50.16667 $\bar{X}_{51}$=56.50000 $\bar{X}_{52}$=59.58333 S_5^2=191.60606 ρ_{51} =0.85239 ρ_{512}= - 0.51542 S_{51}^2=619.72727 ρ_{52} = - 0.86151 K_{51}=0.53380 S_{52}^2=586.62879 K_{52}= - 0.58478 b_{51}=0.47396
VI	$\bar{Y}_6$=41.00000 $\bar{X}_{61}$=46.62500 $\bar{X}_{63}$=39.87500 S_6^2=208.00000 ρ_{61}=0.77808 ρ_{613}=0.71440 S_{61}^2=399.98214 ρ_{63}=0.91753 K_{61}=0.63808 S_{63}^2=496.98214 K_{63}=0.57730 b_{61}=0.56110 b_{63}=0.59358
VII	$\bar{Y}_7$=47.88889 $\bar{X}_{72}$=56.44444 $\bar{X}_{73}$=44.00000

	S_7^2=252.61111	ρ_{72}= - 0.89132	$\rho_{723=}$ - 0.84742
	$S_{72=}^2$502.52778	ρ_{73}=0.92570	K_{72}= - 0.74484
	$S_{73=}^2$384.50000	K_{73}=0.68939	b_{73}=0.75033
VIII	$\overline{Y}_8$=51.33333	$\overline{X}_{81=}$50.66667	$\overline{X}_{82=}$53.13333
	$\overline{X}_{83}$=27.40000	S_8^2=208.66667	$\rho_{81=}$ 0.89878
	ρ_{812} = - 0.77849	$\rho_{813=}$0.56521	$S_{81=}^2$619.66667
	$\rho_{82=}$ - 0.85967	$\rho_{832=}$ - 0.33473	$\rho_{83=}$0.56918
	$S_{82=}^2$353.26667	K_{81}=0.51478	K_{82}= - 0.68387
	K_{83}=0.53627	$S_{83=}^2$66.97143	b_{81}=0.52156
	b_{83}=1.00469		

Table 2: The population parameters for population given in Appendix II

Stratum	***No. of used Auxiliary variable***	***rat cum prod***		***reg cum prod***	
		|Bias|	***MSE***	***|Bias|***	***MSE***
I	0	-	12.95182	-	12.95182
II	1	0.50021	16.98829	-	6.66120
III	1	1.22189	20.02199	1.22189	20.02199
IV	1	0.45453	11.87920	-	3.84618
V	2	0.04614	2.42434	0.15679	0.51149
VI	2	0.50649	7.49781	-	3.55787
VII	2	0.20681	6.30884	0.24285	3.62545
VIII	3	0.03958	2.35296	0.11294	2.00814

Table 3: The biases and mean square errors for ratio cum product and regression cum product method of estimation in each stratum

are presented in the following table:

Estimators	***| Bias |***	***MSE***	***Relative Efficiency***
$\overline{y}$	-	2.76915	100
$\overline{y}_{pr.ratcp}$	0.03931	1.26236	219.36
$\overline{y}_{pr.regcp}$	0.09659	0.84617	327.26

Table 4: The biases, mean square errors and the relative efficiencies of all the estimators of the suggested class

7. Discussion and Conclusion

The proposed estimators using incomplete multi-auxiliary information has been compared with simple mean per unit estimator in which auxiliary information has not been used. It is seen that both proposed estimators are more efficient for mean estimation.

(i) A critical review of table 4 reveals that, though both the suggested estimators are biased but the amount of bias is almost negligible in both the cases.

(ii) When we compare the *V* $(\overline{y})$ with all the proposed estimators, we find that *V*$(\overline{y})$ *= 2.76915*, which is considerably higher than the *MSE* $(\overline{y}_{pr.ratcp})$ *= 1.26236 and MSE* $(\overline{y}_{pr.regcp})$ *=0.84617*.

(iii) The relative efficiency of $\overline{y}_{pr.regcp}$ is highest i.e. 327.26 %. as compared to mean per unit.

It is evident from the above results that the proposed estimators establish the supremacy over $\overline{y}$ in the estimation of mean using stratification on the basis of available incomplete auxiliary information. Thus, we see that how the maximum use of available incomplete multi-auxiliary information can increase the efficiency of the estimators.

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Appendix I

Data Set

Stratum I

Y	36	72	54	35	47	67	38	45	57
	30	26	44	49	67	27	39		

Stratum II

Y	30	26	44	72	40	67	72	43	36
	72	54	35	40					
X_1	18	16	39	95	30	69	80	50	53
	97	28	48	49					

Stratum III

Y	43	36	72	54	35	28	67
X_2	46	55	24	49	72	97	39

Stratum IV

Y	47	67	38	30	26	44	72	40	67
	72								
X_3	29	77	28	20	19	54	65	30	69
	75								

Stratum V

Y	72	43	36	72	54	35	40	34	48
	52	49	67						
X_1	75	19	37	97	53	26	49	44	53
	56	72	92						
X_2	25	41	84	31	56	80	83	96	55
	47	81	36						

Stratum VI

Y	27	39	57	28	36	67	44	30
X_1	18	53	60	34	65	69	54	20
X_3	23	29	60	12	46	82	34	33

Stratum VII

Y	28	57	39	27	67	49	52	40	72
X_2	81	25	72	82	40	72	44	64	28
X_3	12	62	30	23	58	57	55	33	66

Stratum VIII

Y	47	67	38	29	57	44	72	40	63
	69	43	36	72	54	39			
X_1	48	51	24	19	42	54	95	38	69
	75	31	25	97	63	29			
X_2	58	31	74	79	29	41	31	71	53
	46	67	78	29	42	68			
X_3	29	29	16	18	18	20	34	18	31
	39	32	21	35	31	40			