

COMPARISON OF DEGREE OF RANDOMNESS OF THE TABLES OF RANDOM NUMBERS DUE TO TIPPET, FISHER & YATES, KENDALL & SMITH AND RAND CORPORATION

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Abstract

An study has here been made on examining the degree of randomness of the four tables of random numbers due to Tippet, Fisher & Yates, Kendall & Smith and Rand Corporation respectively. Frequency test (based on Chi-square statistic) has been applied in examining the randomness of the four tables. This paper describes the testing of randomness of the four random numbers tables and a comparison of the degree of randomness of them.

Key Words: Random Numbers Table, Tippet, Fisher & Yates, Kendall & Smith, Rand Corporation, Testing of Randomness, Frequency Test.

1. Introduction

Drawing of random sample has been found to be the vital or basic work in every branch of experimental sciences. The scientific method of selecting a random sample consists of the use of random number table. Several tables of random numbers have already been constructed by the renowned researchers. Some of them are (in chronological order) due to Tippett (1927), Mahalanobis (1934), Kendall & Smith (1938, 1939), Fisher & Yates (1938), Hald (1952), Royo & Ferrer (1954), Rand Corporation (1955), Quenouille (1959), Moses & Oakford (1963), Rao, Mitra & Matthai (1966), Snedecor and Cochran (1967), Rohlf & Sokal (1969), Manfred (1971), Hill & Hill (1977) etc. Among these tables, the four tables namely

- (1) Tippett's Random Numbers Table that consists of 10,400 four-digit numbers
- (2) Fisher and Yates Random Numbers Table that comprises 7500 two-digit numbers,
- (3) Kendall and Smith's Random Numbers Table (Kendall & Smith, 1938)
- (4) Random Numbers Table by Rand Corporation (Rand Corporation, 1955).

are widely used in drawing of simple random sample (with or without replacement) from a population.

Fisher & Yates obtained the random numbers from the 10th to 19th digits of A.S. Thompson's 20-figure logarithmic tables. In choosing from those digits, an element of randomness was introduced by using playing cards for the selection of half

pages of the tables and of a column between 10th to 19th and finally for allotting these digits to the 50th place in a block. In this case, the question arises whether the method applied in selecting the numbers has made the numbers random. This creates the necessity of determining the degree of randomness of the random numbers table constructed by Fisher and Yates. Similarly, there is necessity of examining the degree of randomness of the other tables of random numbers. A study has here been made on examining the degree of randomness of the four tables of random numbers due to Tippet, Fisher & Yates, Kendall & Smith and Rand Corporation respectively. Frequency test (based on Chi-square statistic) has been applied in examining the randomness of the four tables. This paper describes the testing of randomness of the four random numbers tables and a comparison of the degree of randomness of them.

2. The Test Statistic Used

Pearson's chi-square test {Yates (1934), Corder & Foreman (2014), Greenwood & Nikulin (1996), Chernoff & Lehmann (1954), Plackett (1983) et al} is used to test the goodness of fit. A test of **goodness of fit** establishes whether or not an observed frequency distribution differs from a theoretical distribution.

The procedure of the test includes the following steps:

1. Calculate the chi-square test statistic χ^2 , which resembles a normalized sum of squared deviations between observed and theoretical frequencies.
2. Determine the degrees of freedom of that statistic, which is essentially the number of categories reduced by the number of parameters of the fitted distribution.
3. Select a desired level of confidence (significance level, p-value or alpha level) for the result of the test.
4. Compare χ^2 to the critical value from the chi-square distribution with degrees of freedom and the selected confidence level (one-sided since the test is only one direction, i.e. is the test value greater than the critical value), which in many cases gives a good approximation of the distribution of χ^2 .
5. Accept or reject the null hypothesis that the observed frequency distribution is different from the theoretical distribution based on whether the test statistic exceeds the critical value of χ^2 . If the test statistic exceeds the critical value of χ^2 , the null hypothesis (H_0 : There is no difference between the distributions) can be rejected with the selected level of confidence and the alternative hypothesis (H_1 : there is a difference between the distributions) can be accepted.

The chi-square test, when used with the standard approximation that a chi-square distribution is applicable, has the following assumptions:

Assumption - 1 (Simple random sample): The sample data is a random sample from a fixed distribution or population where every collection of members of the population of the given sample size has an equal probability of selection. Variants of the test have been developed for complex samples, such as where the data are weighted. Other forms

can be used such as purposive sampling (Bagdonavicius & Nikulin (2011) , Plackett (1983) et al).

Assumption - 2 (Sample size): A sample with a sufficiently large size is assumed. If a chi square test is conducted on a sample with a smaller size, then the chi square test will yield an inaccurate inference. The researcher, by using chi square test on small samples, might end up committing a Type II error (Plackett (1983) , Yates (1934) et al).

Assumption - 3 (Expected cell count): Regarding adequacy of expected cell counts, we assume the following. Some require 5 or more, and others require 10 or more. A common rule is 5 or more in all cells of a 2-by-2 table, and 5 or more in 80% of cells in larger tables, but no cells with zero expected count. When this assumption is not met, Yates's correction is applied (Plackett (1983) , Yates (1934) et al).

Assumption - 4 (Independence): The observations are always assumed to be independent of each other which means that chi-square cannot be used to test correlated data (Plackett, 1983).

Chi-square statistic for the current study

Let us consider Fisher and Yates random number table. This table consists of a total of 15000 digits comprising of 7500 two-digit numbers namely

00 , 01 , 02 , , 98 , 99

The test required in this study is to test whether the occurrences of the numbers appeared in the table is random. This is equivalent to a test to be looked to make sure that equal numbers of 0s, 1s, 2s, 3s, , 9s are present in the table.

Let the number of occurrences of the ten digits in the table be N .

Let

O_i = Observed frequency of the digit i

& E_i = Expected frequency of the digit i

($i = 0 , 1 , 2 , , 9$)

among the N occurrences.

Then the χ^2 statistic for testing the null hypothesis

“the occurrences of the digits in the table is random”

i.e. “each digit has the probability 0.1 to occur in any position”

which is equivalent to testing of the null hypothesis that “the discrepancy between the observed frequencies and the corresponding expected frequencies of the digits is insignificant” is

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

which follows χ^2 distribution with 9 degrees of freedom.

This statistic can be applied to test the randomness of the whole table or of any part of the table. This statistic can similarly be applied in testing of the randomness of the other three tables namely Tippet’s Random Numbers Table, Kendall & Smith’s Random Numbers Table & Random Numbers Table due to Rand Corporation. It is mentioned here that the assumptions under which chi-square test is applicable hold good in the case of the occurrences of the numbers in each of the four tables.

3. Findings of the Tests

The frequency test has been applied to
 1st 100 occurrences , 1st 200 occurrences , 1st 300 occurrences ,
 , all the occurrences
 of the ten digits in the respective table in the case of each of the four random numbers
 tables.

3.1 Findings in case of Fisher & Yates Random Numbers Table

The observed values of chi-square statistic obtained from Fisher & Yates Random Numbers Table have been shown in Table 5.1. In Table-5.1, it is observed that the highest observed value of chi-square with 9 degrees of freedom is 26.118. The theoretical value of chi-square with 9 degrees of freedom at 5% and 1% levels of significance are 16.919 and 21.666 respectively. Thus, the lack of randomness of Fisher & Yates Random Numbers Table can be regarded as significant not only at 5% level of significance but also at 1% level of significance.

However, the observed value of chi-square corresponds to its theoretical value at 0.055% level of significance. Thus, the lack of randomness of Fisher & Yates Random Numbers Table can be regarded as significant at the level of significance $> 0.055\%$ and insignificant at the level of significance $\leq 0.055\%$.

3.2 Findings of the Test for Tippet's Random Numbers Table

The observed values of chi-square statistic obtained from Tippet's Random Numbers Table have been shown in Table 5.2. In Table-5.2, it is observed that the highest observed value of chi-square with 9 degrees of freedom is 15.814. The theoretical value of chi-square with 9 degrees of freedom at 5% level of significance is 16.919. Thus, the lack of randomness of Tippet's Random Numbers Table can be regarded as insignificant at 5% level of significance.

However, the observed value of chi-square corresponds to its theoretical value at 7.5% level of significance. Thus, the lack of randomness of Tippet's Random Numbers Table can be regarded as significant at the level of significance $> 7.5\%$ and insignificant at the level of significance $\leq 7.5\%$.

3.3 Findings of the Test for Kendall & Smith's Random Numbers Table

The observed values of chi-square statistic obtained from Kendall & Smith's Random Numbers Table have been shown in Table 5.3. In Table-5.3, it is observed that the highest observed value of chi-square with 9 degrees of freedom is 13.4 which is less than the corresponding theoretical value of chi-square at 5% level of significance. Thus, the lack of randomness of Kendall & Smith's Random Numbers Table can be regarded as insignificant at 5% level of significance.

However, the observed value of chi-square with 9 degrees of freedom namely 13.4 corresponds to the theoretical value of chi-square with 9 degrees of freedom at 18.1% level of significance. Thus, the lack of randomness of Kendall & Smith's Random Numbers Table can be regarded as significant at the level of significance $> 18.1\%$ and insignificant at the level of significance $\leq 18.1\%$.

3.4 Findings of the Test for Random Numbers Table due to Rand Corporation

The observed values of chi-square statistic obtained from the Random Numbers Table due to Rand Corporation have been shown in Table 5.4. In Table-5.4, it is observed that the highest observed value of chi-square with 9 degrees of freedom is 12.518 which is less than the corresponding theoretical value of chi-square at 5% level of significance. Thus, the lack of randomness of Random Numbers Table due to Rand Corporation can be regarded as insignificant at 5% level of significance.

However, the observed value of chi-square with 9 degrees of freedom namely 12.518 corresponds to the theoretical value of chi-square with 9 degrees of freedom at 24% level of significance. Thus, the lack of randomness of Random Numbers Table due to Rand Corporation can be regarded as significant at the level of significance $>24\%$ and insignificant at the level of significance $\leq 24\%$.

4. Conclusion

From the findings obtained in section 3.4, the degree of the lack of randomness is highest (in other words, the degree of randomness is lowest) in the Fisher & Yates Random Numbers Table among the four tables of random numbers examined. The four random number tables can be ranked with respect to the degree of randomness as given in Table 1.

Name of the Random Numbers Table	Rank with respect to the Degree of Randomness
Due to Fisher & Yates	4
Due to Tippet	3
Due to Kendall & Smith	2
Due to Rand Corporation	1

Table 1: Ranks of tables of random numbers with respect to degree of randomness

One problem for researcher, at this stage, is to make attempt of constructing new random numbers table with more degree of randomness than that of the existing ones. Further, the values of chi-square statistic for different number of digits for the abovesaid four random number tables are given Tables 2, 3, 4 and 5 respectively.

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
100	15.4	4100	4.4239	8100	4.8868	12100	5.4714
200	17.7	4200	3.8658	8200	4.6364	12200	6.1468
300	18.386	4300	4.2776	8300	4.1867	12300	5.469
400	13.1	4400	4.0041	8400	3.5018	12400	7.8212
500	6.28	4500	3.7807	8500	3.072	12500	8.2478
600	10.008	4600	7.2959	8600	3.2025	12600	8.509
700	6.55	4700	2.4961	8700	3.7743	12700	8.8626
800	6.75	4800	3.245	8800	4.5071	12800	8.3969
900	6.189	4900	2.898	8900	3.9144	12900	8.9979
1000	5.48	5000	2.7952	9000	4.248	13000	9.469
1100	7.95	5100	3.589	9100	4.2746	13100	8.8581
1200	7.076	5200	5.777	9200	4.4044	13200	9.664
1300	8.682	5300	3.6998	9300	5.6684	13300	9.766
1400	7.995	5400	3.8445	9400	5.813	13400	8.4756
1500	7.644	5500	3.6058	9500	5.8473	13500	9.497
1600	7.961	5600	5.0036	9600	5.9722	13600	6.5339
1700	8.2808	5700	5.7851	9700	4.8202	13700	9.3647
1800	8.702	5800	6.3243	9800	5.7298	13800	8.817
1900	7.714	5900	7.0341	9900	5.0662	13900	8.5059
2000	9.97	6000	6.688	10000	5.432	14000	8.2736
2100	9.014	6100	7.3731	10100	4.992	14100	8.566
2200	8.297	6200	7.4613	10200	4.8175	14200	9.5832
2300	8.994	6300	7.3573	10300	4.4821	14300	10.449
2400	10.2405	6400	7.279	10400	3.8945	14400	10.526
2500	11.104	6500	6.636	10500	3.872	14500	10.192
2600	9.9448	6600	6.2479	10600	4.0281	14600	9.941
2700	8.541	6700	5.7576	10700	4.677	14700	9.987
2800	6.999	6800	3.5674	10800	4.594	14800	9.725
2900	4.538	6900	5.406	10900	4.203	14900	8.614
3000	6.604	7000	3.4273	11000	3.9865	15000	26.118
3100	4.647	7100	5.084	11100	4.3925		
3200	3.884	7200	5.203	11200	4.651		
3300	4.066	7300	3.047	11300	4.5079		
3400	4.6707	7400	5.1216	11400	4.0389		
3500	4.9755	7500	6.071	11500	3.8884		
3600	3.7949	7600	5.544	11600	4.2668		
3700	3.358	7700	4.8017	11700	5.293		
3800	3.1901	7800	6.1348	11800	6.614		
3900	4.2056	7900	5.2715	11900	5.7344		
4000	4.9175	8000	5.504	12000	5.671		

Table 2: Observed values of χ^2 Statistic for different number of digits of Fisher & Yates Random Number Table

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
100	3.4	4100	8.4865	8100	7.3679	12100	5.287
200	7.9	4200	8.2709	8200	8.251	12200	5.052
300	8.1923	4300	7.7256	8300	7.6912	12300	5.073
400	11.25	4400	10.6277	8400	7.567	12400	5.608
500	14.8	4500	9.53	8500	6.9876	12500	6.32
600	15.6017	4600	8.027	8600	7.2584	12600	6.7166
700	8.7705	4700	9.932	8700	7.7045	12700	7.1556
800	5.8	4800	10.103	8800	7.4427	12800	7.4749
900	10.5318	4900	11.369	8900	7.896	12900	7.1319
1000	6.42	5000	11.392	9000	7.8026	13000	6.3431
1100	11.398	5100	10.5675	9100	7.4145	13100	6.0259
1200	13.1479	5200	11.1768	9200	8.5159	13200	5.5684
1300	10.9994	5300	11.4629	9300	8.1087	13300	5.7254
1400	10.0284	5400	10.5303	9400	9.3894	13400	6.3911
1500	9.9269	5500	10.4275	9500	10.404	13500	11.8595
1600	8.8126	5600	10.5318	9600	9.7165	13600	4.9459
1700	9.5345	5700	11.3367	9700	9.9068	13700	4.7411
1800	9.398	5800	15.814	9800	9.8328	13800	3.5648
1900	9.4842	5900	10.135	9900	8.9808	13900	4.885
2000	8.4	6000	10.805	10000	8.296	14000	12.3586
2100	6.9248	6100	9.8492	10100	9.2524	14100	4.9539
2200	4.5816	6200	9.5807	10200	8.2079	14200	4.4883
2300	4.3787	6300	9.4377	10300	8.0387	14300	4.3462
2400	4.0041	6400	10.669	10400	8.1328	14400	4.1505
2500	3.664	6500	9.3656	10500	8.591	14500	6.1549
2600	5.3768	6600	10.458	10600	9.4579	14600	4.4307
2700	4.2889	6700	10.372	10700	9.9099	14700	4.1109
2800	4.6569	6800	8.3655	10800	9.4313	14800	3.449
2900	5.7929	6900	8.6899	10900	8.2084	14900	3.449
3000	5.6331	7000	8.0287	11000	6.5808	15000	3.1974
3100	6.0767	7100	7.8962	11100	6.6414	15100	3.094
3200	6.4124	7200	8.6185	11200	5.848	15200	3.617
3300	6.7509	7300	8.9613	11300	5.5923	15300	3.3507
3400	6.8759	7400	8.9986	11400	5.0608	15400	3.3773
3500	8.5087	7500	8.333	11500	4.484	15500	3.3915
3600	6.6126	7600	9.1746	11600	4.869	15600	2.994
3700	7.2001	7700	9.699	11700	5.0943	15700	3.3114
3800	7.5998	7800	8.7332	11800	4.536	15800	2.3523
3900	7.3331	7900	9.2667	11900	4.531	15900	3.182
4000	8.335	8000	8.7695	12000	4.3896	16000	3.2736

Table Contd.....

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
16100	2.9751	20100	3.203	24100	2.326	28100	4.4433
16200	2.3912	20200	3.637	24200	2.2385	28200	4.5722
16300	2.7744	20300	3.8863	24300	2.162	28300	3.9653
16400	2.5864	20400	3.65	24400	2.3401	28400	3.9282
16500	2.4243	20500	3.789	24500	2.7055	28500	4.0976
16600	2.2081	20600	3.817	24600	2.821	28600	4.4385
16700	2.321	20700	3.759	24700	2.8491	28700	5.487
16800	2.5904	20800	4.119	24800	3.5152	28800	4.1992
16900	1.8203	20900	4.388	24900	2.6747	28900	4.258
17000	1.885	21000	4.692	25000	3.3152	29000	4.6386
17100	1.5281	21100	3.752	25100	3.8342	29100	4.3189
17200	1.5152	21200	4.1596	25200	3.9595	29200	4.1495
17300	1.6973	21300	4.0916	25300	4.098	29300	4.6732
17400	1.6859	621400	3.848	25400	4.3545	29400	4.6489
17500	1.537	21500	3.926	25500	4.1754	29500	5.2756
17600	1.546	21600	3.724	25600	3.921	29600	4.9919
17700	1.6856	21700	5.021	25700	4.3292	29700	4.9808
17800	1.546	21800	3.382	25800	4.5341	29800	5.2645
17900	2.3487	21900	3.224	25900	4.3937	29900	5.4562
18000	1.8919	22000	3.3899	26000	4.4678	30000	6.0744
18100	2.282	22100	3.5105	26100	4.4131	30100	5.9296
18200	2.313	22200	3.1577	26200	4.6464	30200	6.1662
18300	2.549	22300	3.5166	26300	4.7937	30300	5.8754
18400	2.4539	22400	3.5672	26400	4.7704	30400	6.4819
18500	2.3757	22500	3.3201	26500	4.7163	30500	6.1508
18600	2.4022	22600	3.0983	26600	4.7927	30600	6.3709
18700	2.6152	22700	2.9388	26700	4.6195	30700	6.3003
18800	2.157	22800	3.153	26800	4.5859	30800	6.5163
18900	2.656	22900	2.9298	26900	4.5969	30900	6.7915
19000	3.2563	23000	2.9128	27000	4.9096	31000	6.4007
19100	2.377	23100	2.574	27100	4.9112	31100	6.6209
19200	2.714	23200	2.5741	27200	4.4823	31200	6.5731
19300	2.468	23300	2.77129	27300	4.667	31300	6.6937
19400	2.802	23400	2.3837	27400	4.8887	31400	6.8152
19500	3.138	23500	2.4918	27500	4.727	31500	6.901
19600	2.989	23600	2.2985	27600	4.7702	31600	6.7705
19700	2.864	23700	2.2616	27700	4.7248	31700	6.9166
19800	3.0296	23800	2.4785	27800	4.8596	31800	6.307
19900	2.912	23900	2.274	27900	4.7382	31900	6.7621
20000	2.798	24000	2.2924	28000	4.7695	32000	6.1931

Table Contd.....

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
32100	6.0795	36100	11.2016	40100	8.6943
32200	6.0855	36200	11.3414	40200	8.6632
32300	6.0664	36300	11.5442	40300	8.921
32400	5.8082	36400	11.3204	40400	9.0153
32500	6.07	36500	11.3343	40500	9.100
32600	6.257	36600	11.4013	40600	8.8044
32700	6.2384	36700	11.4653	40700	9.3351
32800	6.4827	36800	11.7771	40800	9.1598
32900	6.0148	36900	11.4582	40900	9.5824
33000	6.5653	37000	11.6292	41000	9.3375
33100	6.6835	37100	11.0308	41100	9.5207
33200	7.1698	37200	11.0016	41200	9.866
33300	7.7898	37300	10.2003	41300	9.9955
33400	7.9719	37400	10.289	41400	9.9817
33500	7.7937	37500	9.3559	41500	10.1614
33600	8.5374	37600	9.825	41600	10.3015
33700	8.9082	37700	9.7166		
33800	9.5029	37800	9.4323		
33900	9.2077	37900	9.9624		
34000	9.4735	38000	9.076		
34100	9.0944	38100	8.8845		
34200	8.7398	38200	8.744		
34300	8.6688	38300	8.3849		
34400	8.8238	38400	9.3174		
34500	8.7549	38500	9.0419		
34600	9.7251	38600	8.9897		
34700	9.9682	38700	8.8835		
34800	9.6288	38800	8.692		
34900	10.2195	38900	8.278		
35000	9.9896	39000	8.1621		
35100	10.6078	39100	8.722		
35200	9.9749	39200	8.3996		
35300	9.8425	39300	8.845		
35400	8.1661	39400	9.1019		
35500	10.2705	39500	8.8997		
35600	10.3578	39600	8.8236		
35700	10.2734	39700	9.2405		
35800	10.2379	39800	9.5332		
35900	10.0995	39900	9.1102		
36000	9.7082	40000	9.1902		

Table 3 : Observed values of χ^2 Statistic for different number of digits of Tippett's Random Number Table

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
100	5.6	4100	4.312	8100	6.998	12100	7.427
200	13.4	4200	4.262	8200	6.919	12200	7.308
300	7.265	4300	5.1503	8300	6.461	12300	7.215
400	9.75	4400	5.2983	8400	6.956	12400	6.288
500	10.16	4500	5.858	8500	8.74	12500	6.32
600	12.769	4600	5.321	8600	9.522	12600	6.30
700	11.687	4700	4.077	8700	9.523	12700	6.485
800	11.226	4800	4.013	8800	10.568	12800	7.283
900	10.267	4900	5.245	8900	9.971	12900	8.566
1000	7.84	5000	5.484	9000	9.841	13000	7.529
1100	9.417	5100	5.783	9100	10.378	13100	7.889
1200	7.206	5200	6.501	9200	10.162	13200	7.999
1300	6.7687	5300	5.412	9300	9.965	13300	8.239
1400	5.643	5400	5.036	9400	9.531	13400	7.153
1500	7.026	5500	5.388	9500	9.404	13500	6.262
1600	5.174	5600	5.462	9600	9.039	13600	6.574
1700	4.882	5700	4.681	9700	9.421	13700	6.998
1800	4.167	5800	4.092	9800	9.067	13800	6.755
1900	3.147	5900	4.307	9900	9.419	13900	6.64
2000	2.43	6000	5.362	10000	9.391	14000	5.635
2100	2.04	6100	6.423	10100	9.03	14100	5.127
2200	2.583	6200	6.395	10200	9.682	14200	4.769
2300	2.565	6300	5.806	10300	10.189	14300	4.736
2400	4.225	6400	6.675	10400	9.105	14400	4.918
2500	3.768	6500	6.774	10500	8.75	14500	5.397
2600	3.7144	6600	6.649	10600	8.944	14600	5.412
2700	4.628	6700	7.77	10700	8.066	14700	5.507
2800	2.971	6800	8.027	10800	7.659	14800	5.161
2900	3.00	6900	6.892	10900	8.427	14900	5.246
3000	2.878	7000	6.206	11000	8.571	15000	5.111
3100	2.687	7100	7.088	11100	8.451	15100	5.608
3200	2.887	7200	6.621	11200	9.253	15200	5.289
3300	2.077	7300	6.239	11300	8.567	15300	4.314
3400	2.695	7400	6.565	11400	7.441	15400	4.554
3500	2.69	7500	6.193	11500	7.708	15500	4.276
3600	3.071	7600	6.524	11600	8.216	15600	3.483
3700	2.944	7700	7.528	11700	8.937	15700	3.261
3800	3.201	7800	7.336	11800	8.396	15800	3.445
3900	4.498	7900	7.863	11900	7.337	15900	3.39
4000	4.4715	8000	7.771	12000	6.824	16000	3.732

Table Contd.....

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
16100	3.492	20100	3.061	24100	3.766	28100	2.742
16200	3.327	20200	3.582	24200	3.858	28200	2.797
16300	2.836	20300	3.714	24300	3.968	28300	2.737
16400	3.04	20400	3.89	24400	4.298	28400	2.721
16500	3.171	20500	3.769	24500	4.698	28500	2.929
16600	3.562	20600	3.629	24600	4.556	28600	2.989
16700	3.433	20700	3.657	24700	4.24	28700	3.188
16800	3.137	20800	3.635	24800	3.772	28800	3.036
16900	3.536	20900	3.84	24900	3.961	28900	3.071
17000	3.955	21000	3.634	25000	4.32	29000	3.387
17100	3.93	21100	3.978	25100	4.372	29100	3.46
17200	4.226	21200	4.074	25200	4.155	29200	3.45
17300	4.004	21300	3.962	25300	3.943	29300	3.88
17400	4.045	21400	3.96	25400	3.903	29400	3.537
17500	4.129	21500	3.824	25500	3.77	29500	3.427
17600	4.501	21600	3.607	25600	3.475	29600	3.61
17700	4.771	21700	3.65	25700	3.281	29700	3.612
17800	4.868	21800	3.445	25800	3.35	29800	3.662
17900	4.318	21900	3.401	25900	3.138	29900	3.428
18000	4.837	22000	3.577	26000	3.16	30000	3.32
18100	5.262	22100	3.576	26100	3.164	30100	3.396
18200	5.083	22200	4.393	26200	3.23	30200	3.773
18300	4.958	22300	4.498	26300	2.966	30300	3.936
18400	4.589	22400	3.791	26400	2.833	30400	4.124
18500	4.312	22500	4.632	26500	2.582	30500	4.082
18600	3.887	22600	3.832	26600	3.012	30600	4.433
18700	3.72	22700	3.957	26700	3.044	30700	4.656
18800	3.812	22800	4.306	26800	2.878	30800	4.51
18900	4.079	22900	5.351	26900	2.69	30900	4.206
19000	3.855	23000	3.543	27000	2.466	31000	4.048
19100	3.852	23100	3.185	27100	2.265	31100	4.028
19200	4.476	23200	3.669	27200	2.534	31200	4.26
19300	3.652	23300	3.833	27300	2.4009	31300	4.209
19400	3.791	23400	3.649	27400	2.554	31400	4.2
19500	3.256	23500	3.713	27500	2.558	31500	4.47
19600	3.921	23600	3.57	27600	2.65	31600	4.224
19700	3.474	23700	3.52	27700	2.60	31700	4.634
19800	3.346	23800	3.734	27800	2.66	31800	5.018
19900	3.454	23900	4.15	27900	2.92	31900	5.05
20000	3.203	24000	4.169	28000	2.496	32000	4.709

Table Contd.....

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
32100	4.987	36100	5.362	40100	7.914	44100	8.854
32200	4.573	36200	5.592	40200	7.973	44200	8.695
32300	4.93	36300	5.763	40300	8.182	44300	8.88
32400	5.061	36400	6.084	40400	8.318	44400	8.656
32500	4.87	36500	5.674	40500	8.229	44500	8.769
32600	4.766	36600	5.722	40600	8.574	44600	9.059
32700	4.927	36700	5.505	40700	8.469	44700	9.463
32800	5.44	36800	5.522	40800	7.919	44800	9.522
32900	5.315	36900	5.726	40900	7.80	44900	9.91
33000	4.881	37000	5.853	41000	8.034	45000	10.16
33100	5.213	37100	6.11	41100	8.636	45100	10.272
33200	5.588	37200	5.94	41200	8.604	45200	10.38
33300	5.998	37300	6.06	41300	8.992	45300	10.18
33400	4.898	37400	6.401	41400	9.303	45400	10.378
33500	5.328	37500	6.129	41500	9.694	45500	10.54
33600	5.384	37600	5.90	41600	10.04	45600	10.66
33700	5.127	37700	6.15	41700	9.605	45700	9.959
33800	5.056	37800	5.884	41800	9.391	45800	9.52
33900	5.382	37900	6.371	41900	9.021	45900	10.004
34000	4.88	38000	6.317	42000	8.861	46000	9.91
34100	5.014	38100	6.34	42100	8.972	46100	9.95
34200	4.962	38200	6.838	42200	9.014	46200	9.161
34300	5.245	38300	6.757	42300	8.921	46300	9.416
34400	5.517	38400	6.731	42400	8.714	46400	9.42
34500	5.094	38500	6.871	42500	8.756	46500	9.368
34600	5.408	38600	6.911	42600	8.724	46600	9.598
34700	5.875	38700	7.025	42700	8.824	46700	9.94
34800	6.093	38800	6.686	42800	8.627	46800	9.372
34900	6.172	38900	6.98	42900	8.536	46900	9.555
35000	6.114	39000	5.942	43000	8.514	47000	9.754
35100	6.065	39100	6.994	43100	8.66	47100	9.147
35200	5.793	39200	7.599	43200	8.23	47200	9.308
35300	5.662	39300	7.774	43300	8.259	47300	9.844
35400	5.487	39400	7.779	43400	8.465	47400	9.473
35500	6.06	39500	7.616	43500	8.854	47500	9.279
35600	6.084	39600	7.837	43600	8.663	47600	10.207
35700	5.928	39700	7.965	43700	9.197	47700	10.692
35800	5.734	39800	7.967	43800	8.634		
35900	5.832	39900	7.868	43900	8.717		
36000	5.516	40000	7.829	44000	8.862		

Table 4: Observed values of χ^2 Statistic for different number of digits of Kendall & Smith's Random Number Table

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
100	5.7	4500	1.848	8900	7.345	13300	8.849
200	11.6	4600	2.288	9000	6.473	13400	10.001
300	9.52	4700	2.267	9100	6.729	13500	9.787
400	6.416	4800	3.216	9200	7.748	13600	10.303
500	8.52	4900	2.726	9300	8.078	13700	10.039
600	6.772	5000	3.136	9400	7.944	13800	9.542
700	6.285	5100	2.709	9500	7.761	13900	9.567
800	7.202	5200	2.776	9600	9.219	14000	9.076
900	7.967	5300	3.558	9700	9.074	14100	9.158
1000	10.36	5400	3.457	9800	9.615	14200	9.466
1100	7.289	5500	3.874	9900	9.217	14300	8.419
1200	6.014	5600	3.933	10000	8.442	14400	7.866
1300	6.614	5700	3.841	10100	9.132	14500	7.696
1400	9.729	5800	4.438	10200	8.45	14600	8.59
1500	7.815	5900	4.608	10300	8.669	14700	8.01
1600	6.961	6000	4.843	10400	9.223	14800	7.612
1700	8.858	6100	5.587	10500	8.702	14900	8.008
1800	8.798	6200	7.04	10600	8.935	15000	8.463
1900	7.62	6300	6.821	10700	8.911	15100	7.927
2000	7.81	6400	5.381	10800	9.49	15200	7.82
2100	6.905	6500	6.681	10900	9.615	15300	7.388
2200	6.138	6600	5.907	11000	8.815	15400	6.506
2300	5.251	6700	5.647	11100	8.933	15500	6.058
2400	4.842	6800	5.77	11200	9.187	15600	7.094
2500	4.976	6900	6.889	11300	9.346	15700	7.338
2600	5.13	7000	7.066	11400	9.599	15800	7.343
2700	5.295	7100	5.961	11500	9.912	15900	6.839
2800	5.871	7200	5.696	11600	10.023	16000	6.88
2900	5.889	7300	4.827	11700	9.838	16100	6.991
3000	6.544	7400	5.289	11800	9.102	16200	6.323
3100	6.157	7500	4.659	11900	9.321	16300	6.673
3200	6.363	7600	5.51	12000	9.573	16400	6.565
3300	3.829	7700	5.732	12100	10.79	16500	6.777
3400	3.954	7800	5.661	12200	9.612	16600	7.668
3500	3.206	7900	6.885	12300	9.704	16700	7.3682
3600	2.832	8000	6.774	12400	8.875	16800	7.319
3700	0.024	8100	6.72	12500	9.192	16900	7.578
3800	2.395	8200	7.784	12600	9.843	17000	7.637
3900	2.303	8300	7.071	12700	9.517	17100	7.773
4000	1.738	8400	6.095	12800	8.926	17200	8.28
4100	1.668	8500	6.519	12900	8.926	17300	7.513
4200	2.484	8600	7.955	13000	8.333	17400	6.96
4300	1.962	8700	8.123	13100	8.057	17500	6.765
4400	1.394	8800	8.805	13200	8.389	17600	6.758

Table Contd.....

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
17700	7.219	21900	8.504	26100	11.307	30300	11.387
17800	7.76	22000	8.416	26200	9.703	30400	11.942
17900	7.859	22100	8.729	26300	10.454	30500	12.226
18000	7.851	22200	9.111	26400	9.120	30600	12.248
18100	8.082	22300	9.601	26500	8.994	30700	12.03
18200	8.06	22400	9.256	26600	9.041	30800	11.722
18300	8.19	22500	8.53	26700	9.417	30900	11.464
18400	7.977	22600	8.742	26800	10.228	31000	12.093
18500	7.446	22700	9.347	26900	10.544	31100	12.253
18600	7.512	22800	8.844	27000	10.378	31200	11.696
18700	6.953	22900	8.904	27100	10.433	31300	11.715
18800	7.09	23000	9.227	27200	10.653	31400	11.779
18900	6.965	23100	9.44	27300	10.594	31500	11.965
19000	5.575	23200	9.958	27400	9.15	31600	11.854
19100	7.087	23300	10.081	27500	9.365	31700	11.576
19200	7.228	23400	10.334	27600	9.472	31800	11.642
19300	7.469	23500	10.214	27700	9.817	31900	10.801
19400	7.755	23600	10.232	27800	9.685	32000	10.67
19500	8.146	23700	9.951	27900	9.44	32100	10.111
19600	8.114	23800	9.931	28000	9.274	32200	9.891
19700	7.868	23900	10.213	28100	9.67	32300	9.596
19800	8.371	24000	10.257	28200	9.464	32400	9.681
19900	8.712	24100	10.613	28300	9.512	32500	9.719
20000	8.232	24200	10.798	28400	9.413	32600	9.27
20100	7.638	24300	9.958	28500	9.738	32700	9.129
20200	8.312	24400	10.019	28600	10.333	32800	9.113
20300	7.908	24500	10.613	28700	10.829	32900	9.608
20400	7.932	24600	10.778	28800	10.354	33000	9.636
20500	6.835	24700	11.301	28900	9.647	33100	9.82
20600	7.956	24800	11.589	29000	9.335	33200	9.586
20700	7.706	24900	11.111	29100	9.494	33300	9.906
20800	7.956	25000	10.604	29200	9.82	33400	10.065
20900	8.532	25100	10.405	29300	9.217	33500	9.973
21000	9.224	25200	11.133	29400	9.676	33600	10.017
21100	9.031	25300	11.399	29500	9.363	33700	9.799
21200	9.387	25400	11.031	29600	9.431	33800	10.061
21300	9.174	25500	11.336	29700	9.841	33900	10.367
21400	9.5325	25600	11.474	29800	9.973	34000	9.546
21500	8.824	25700	10.904	29900	10.743	34100	9.628
21600	8.916	25800	10.981	30000	9.997	34200	9.559
21700	8.03	25900	10.536	30100	11.192	34300	9.909
21800	8.3197	26000	11.166	30200	11.337	34400	9.438

Table Contd.....

Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2	Number of Digits (1 st)	Value of χ^2
34500	9.444	38500	7.585	42500	7.781	46500	7.145
34600	9.233	38600	7.509	42600	7.802	46600	7.274
34700	10.254	38700	7.57	42700	7.383	46700	7.194
34800	12.518	38800	7.787	42800	7.796	46800	7.319
34900	9.902	38900	7.519	42900	7.362	46900	7.414
35000	9.423	39000	7.379	43000	7.262	47000	7.149
35100	9.44	39100	7.446	43100	7.239	47100	7.431
35200	10.023	39200	7.276	43200	7.171	47200	7.537
35300	10.224	39300	7.42	43300	7.469	47300	7.71
35400	10.068	39400	6.752	43400	7.48	47400	7.826
35500	10.57	39500	7.282	43500	7.465	47500	7.469
35600	9.935	39600	8.175	43600	7.796	47600	7.208
35700	10.063	39700	6.732	43700	7.455	47700	7.537
35800	9.848	39800	6.878	43800	7.309	47800	7.77
35900	10.104	39900	7.026	43900	6.981	47900	7.856
36000	10.257	40000	7.251	44000	6.959	48000	8.435
36100	10.095	40100	7.315	44100	7.127	48100	7.13
36200	9.434	40200	7.405	44200	7.052	48200	7.265
36300	9.136	40300	7.648	44300	7.046	48300	7.42
36400	9.053	40400	7.997	44400	7.177	48400	7.539
36500	8.968	40500	8.034	44500	7.375	48500	7.868
36600	8.919	40600	7.909	44600	7.228	48600	7.498
36700	8.945	40700	8.276	44700	7.286	48700	7.402
36800	8.943	40800	8.359	44800	7.615	48800	7.248
36900	8.72	40900	8.029	44900	8.005	48900	8.189
37000	8.437	41000	7.836	45000	7.693	49000	6.433
37100	8.035	41100	7.529	45100	7.905	49100	6.542
37200	8.167	41200	7.527	45200	7.394	49200	7.339
37300	7.672	41300	7.322	45300	7.458	49300	7.597
37400	7.923	41400	7.695	45400	7.441	49400	7.824
37500	7.614	41500	7.442	45500	7.551	49500	7.779
37600	7.731	41600	7.106	45600	7.417	49600	7.881
37700	7.524	41700	7.535	45700	7.236	49700	7.927
37800	7.375	41800	7.656	45800	7.72	49800	7.623
37900	7.827	41900	7.665	45900	7.352	49900	7.576
38000	7.531	42000	7.574	46000	7.382	50000	7.271
38100	7.327	42100	7.691	46100	7.829		
38200	7.632	42200	7.817	46200	7.649		
38300	7.545	42300	7.921	46300	7.395		
38400	7.403	42400	7.77	46400	7.535		

Table 5: Observed values of χ^2 Statistic for different number of digits of Random Number Table due to Rand Corporation

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